Large-scale inhomogeneities in modified Chaplygin gas cosmologies

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We extend the homogeneous modified Chaplygin cosmologies to large-scale perturbations by formulating a Zeldovich-like approximation. We show that the model interpolates between an epoch with a soft equation of state and a de Sitter phase, and that in the intermediate regime its matter content is simply the sum of dust and a cosmological constant. We then study how the large-scale inhomogeneities evolve and compare the results with cold dark matter (CDM), ΛCDM and generalized Chaplygin scenarios. Interestingly, we find that unlike the latter, our models do always resemble ΛCDM.

PACS numbers: 98.80.Cq

I. INTRODUCTION

According to increasing astrophysical indicia, the evolution of the Universe seems to be largely governed by dark energy with negative pressure together with pressureless cold dark matter (see [1] for the latest review) in a two to one proportion. However, little is know about the origin of either component, which in the standard cosmological model would play very different roles: dark matter would be responsible for matter clustering, whereas dark energy [2] would account for accelerated expansion. Several candidates for dark energy haven proposed and confronted with observations: a purely cosmological constant, quintessence with a single field (see [4] for earliest papers) or two coupled fields [5], k-essence scalar fields, and phantom energy [6]. Interestingly, a bolder alternative presented recently suggests that an effective dark energy-like equation of state could be due to averaged quantum effects [7].

The lack of information regarding the provenance of dark matter and dark energy allows for speculation with the economical and aesthetic idea that a single component acted in fact as both dark matter and dark energy. The unification of those two components has risen a considerable theoretical interest, because on the one hand model building becomes considerable simpler, and on the other hand such unification implies the existence of an era during which the energy densities of dark matter and dark energy are strikingly similar.

One possible way to achieve that unification is through a particular k-essence fluid, the generalized Chaplygin gas [8], with the exotic equation of state

$$p = -\frac{A}{\rho^{\beta}} \tag{1}$$

where constants β and A satisfy respectively $0 < \beta \leq 1$ and $A > 0$. Using the energy conservation equation one obtains an energy density with evolution given by

$$\rho = \left(A + \frac{B}{a^{3(1+\beta)}}\right)^{1/(1+\beta)} , \tag{2}$$

where a is the scale factor and $B > 0$ is an integration constant. This model interpolates between a $\rho \propto a^{-3}$ evolution law at early times and $\rho \simeq$ cons. at late times (i.e. the model is dominated by dust in its early stages and by vacuum energy in its late history). In the intermediate regime the matter content of the model can be approximated by the sum of a cosmological constant an a fluid with a soft equation of state $p = \beta\rho$. The traditional Chaplygin gas [9] corresponds to $\beta = 1$ (stiff equation of state).

Another possibility which has emerged recently is the modified Chaplygin gas [10]. Its equation of state is

$$p = \frac{1}{\alpha - 1}\left(\rho - \alpha A \rho^{\frac{1}{\alpha}}\right), \tag{3}$$

with $\alpha > 0$ a constant. Integrating the conservation equation for an homogeneous and isotropic spacetime

$$\dot{\rho} + 3H(p + \rho) = 0 , \tag{4}$$

where $H = \dot{a}/a$ is the expansion rate of the Universe and dots represent differentiation with respect to cosmic time, we get the energy density

$$\rho = \left(A + \frac{B}{a^3}\right)^{\frac{\alpha}{\alpha-1}} . \tag{5}$$

Modified Chaplygin cosmologies with $\alpha > 1$ are transient models which interpolate between a $\rho \propto a^{-3\alpha/(\alpha-1)}$ evolution law at early times and a de Sitter phase at late times, but interestingly the matter content at the intermediate stage is a mixture of dust and a cosmological constant. The sound speed for the modified Chaplygin gas [10] becomes

$$c_s^2 = \frac{1}{\alpha - 1}\left[1 - A\rho^{(1-\alpha)/\alpha}\right] . \tag{6}$$

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The observational tests of traditional and generalized Chaplygin models are numerous (but no such study has been carried out with the modified models considered here). According to Wilkinson Microwave Anisotropy Probe (WMAP) generalized Chaplygin models must have a very soft ($p/\rho \leq 0.2$) equation of state in the intermediate regime between dust and dark energy domination [11, 12]. In addition, they are much likelier as dark energy models than as unified dark matter models [12]. However, when CMB data are combined with supernovae data both discrepancies and controversy arise. It turns out that Chaplygin gases are preferred over ΛCDM [13, 14], although the traditional case with a stiff equation of state in the intermediate regime is not the best fit [13] (nevertheless it has been claimed that the current data are not powerful enough to discriminate between stiffness and softness [15]). This is not the end of the story, though, because [16] (which uses supernovae observations) and [17] (which combines SNIa and CMB data) rule out Chaplygin gases as unified dark matter models. As discussed in [14] the reason of the discrepancy with [16] is that not even a small fraction of baryons (which may have a crucial influence) was considered, whereas the dissimilarity with the result in [17] may be traced back to the fact that CMB analysis has inherently a high degeneracy in the space of parameters. Yet to add more confusion it seems that the limits obtained from Cosmic All Sky Survey (CLASS) statistics are only marginally compatible with the ones obtained from other cosmological tests [18].

In this paper we shall be concerned with the evolution of large-scale inhomogeneities in modified Chaplygin cosmologies. This is an issue of interest because candidates for the dark matter and dark matter unification will only be valid if they ensure that initial perturbations can evolve into a deeply nonlinear regime to form a gravitational condensate of super-particles that can act like cold dark matter. Here we will follow the covariant and sufficiently general Zeldovich-like non-perturbative approach given in [19], because it can be adapted to any balometric or parametric equation of state. Our results indicate that our model fits well in the standard structure formation scenarios. Although, in general we find a fairly similar behavior to generalized Chaplygin models [8], it must be pointed out that perturbations in our model either grow or stay at the saturation value like in the ΛCDM matter scenario. In contrast, generalized Chaplygin have the somewhat unwanted feature that for very soft the equations of state there in an epoch during which perturbations decrease and resemblance with the ΛCDM matter scenario ceases temporarily.

II. THE MODEL

For the modified Chaplygin gas described by Eqs. (3) and (5) the effective equation of state in the intermediate regime between the dust dominated phase and the de Sitter phase can be obtained expanding Eqs. (5) and (3) in powers of Ba^{-3}, we get

$$\rho = A^{\frac{\alpha}{\alpha-1}} + A^{\frac{1}{\alpha-1}}\frac{\alpha\, B}{(\alpha-1)a^3} + \mathcal{O}\left(\frac{B^2}{a^6}\right) \tag{7}$$

$$p = -A^{\frac{\alpha}{\alpha-1}} + \mathcal{O}\left(\frac{B^2}{a^6}\right), \tag{8}$$

which corresponds to a mixture of vacuum energy density $A^{\frac{\alpha}{\alpha-1}}$, presureless dust and other perfect fluids which dominate at the very beginning of the universe. In the intermediate regime the modified Chaplygin gas behaves as dust at the time where the energy density satisfies the condition $\rho = A^{\alpha/(\alpha-1)}$. At very early times the equation of state parameter $w \equiv p/\rho$ becomes

$$w \simeq c_s^2 \simeq \frac{1}{\alpha-1}, \tag{9}$$

so that for very large α the dust-like behavior is recovered.

The next step is to investigate what sort of cosmological model arises when we consider a slight inhomogeneous modified Chaplygin cosmologies. For a general metric $g_{\mu\nu}$, the proper time $d\tau = \sqrt{g_{00}}dx^0$, and $\gamma \equiv -g/g_{00}$ as the determinant of the induced 3-metric, one has

$$\gamma_{ij} = \frac{g_{i0}g_{j0}}{g_{00}} - g_{ij}\ . \tag{10}$$

In the first approximation it will be interesting to investigate the contribution of inhomogeneities introduced in the modified Chaplygin gas through the expression

$$\rho = \left(A + \frac{B}{\sqrt{\gamma}}\right)^{\frac{\alpha}{\alpha-1}}\ . \tag{11}$$

The latter result suggests that the evolution of inhomogeneities can be studied using the Zeldovich method through the deformation tensor [19, 20, 21]:

$$D_i^j = a(t)\left(\delta_i^j - b(t)\frac{\partial^2\varphi(\vec{q})}{\partial q^i \partial q^j}\right)\ , \tag{12}$$

where $b(t)$ parametrizes the time evolution of the inhomogeneities and $\vec{q}$ are generalized Lagrangian coordinates so that

$$\gamma_{ij} = \delta_{mn} D_i^m D_j^n\ , \tag{13}$$

and h is a perturbation

$$h = 2b(t)\varphi_{,i}{}^{i}\ . \tag{14}$$

Hence, using the equations above and Eqs. (7) and (8), it follows that

$$\rho \simeq \bar{\rho}(1+\delta)\ , \tag{15}$$

$$p \simeq \frac{1}{\alpha-1}\left(\bar{\rho} - A\,\alpha\,\bar{\rho}^{\,\frac{1}{\alpha}} + \delta\left(\bar{\rho} - A\,\bar{\rho}^{\,\frac{1}{\alpha}}\right)\right)\ , \tag{16}$$

where $\bar{\rho}$ is given by Eq. (5) and the density contrast δ is related to h through

$$\delta = \frac{h}{2}(1+w) \ , \tag{17}$$

where $w \equiv \bar{p}/\bar{\rho}$. Finally, after some algebra we get

$$\bar{p} = \bar{\rho}\left(w + \frac{(1+w)\,\delta}{\alpha}\right) . \tag{18}$$

Now, the metric (13) leads to the following 00 component of the Einstein equations:

$$-3\frac{\ddot{a}}{a}+\frac{1}{2}\ddot{h}+H\dot{h} = 4\pi G\bar{\rho}\left(1+3w+\left(1+\frac{3\,(1+w)}{\alpha}\right)\delta\right) \ , \tag{19}$$

where the unperturbed part of this equation corresponds to the Raychaudhuri equation

$$-\,3\frac{\ddot{a}}{a} = 4\pi G\bar{\rho}(1+3w) \ . \tag{20}$$

Using the Friedmann equation for a flat spacetime $H^2 = 8\pi G\bar{\rho}/3$, one can rewrite Eq. (19) as a differential equation for $b(a)$:

$$\frac{2}{3}a^2 b''+(1-w)ab'-(1+w)\left(1+\frac{3\,(1+w)}{\alpha}\right)b = 0 \ , \tag{21}$$

where the primes denote derivatives with respect to the scale-factor, a.

An expression for w as a function of the scale-factor can be derived from Eq. (3):

$$w(a) = \frac{B-(\alpha-1)Aa^3}{(\alpha-1)\,(B+Aa^3)} . \tag{22}$$

The latter must be conveniently recast in terms of the fractional vacuum and matter energy densities. This can be done by using

$$\lim_{\alpha\to\infty}\rho = A + \frac{B}{a^3} \tag{23}$$

combined with

$$H^2 = H_0^2\left(\Omega_{m0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda 0}\right) . \tag{24}$$

where H_0 and a_0 are, respectively, the current value of the Hubble and scale factor, and $\Omega_{\Lambda 0}$ and Ω_{m0} are, respectively, the fractional vacuum and matter energy densities today. Setting $a_0 = 1$ we obtain

$$w(a) = \frac{\Omega_{m0}-(\alpha-1)\Omega_{\Lambda 0}a^3}{(\alpha-1)\,(\Omega_{m0}+\Omega_{\Lambda 0}a^3)} , \tag{25}$$

and consistently

$$\lim_{\alpha\to\infty} w(a) = -\frac{\Omega_{\Lambda 0}a^3}{\Omega_{m0}+\Omega_{\Lambda 0}a^3} . \tag{26}$$

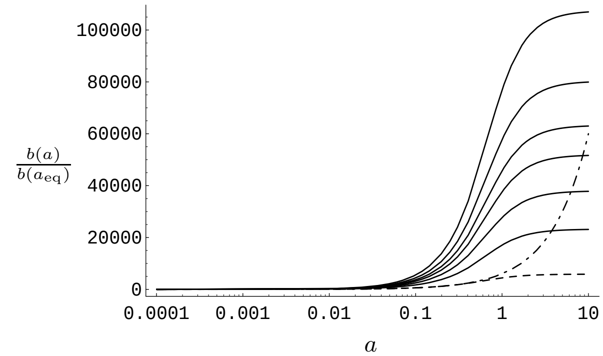

FIG. 1: Evolution of $b(a)/b(a_{\rm eq})$ for the modified Chaplygin gas for $\alpha = 10, 1, 12, 13, 15, 20$ (continuous lines) as compared with ΛCDM (dashed line) and CDM (dashed-dotted line). Lower curves correspond to higher values of α.

We have used this expression to integrate Eq. (21) numerically, for different values of α, and taking $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ [22]. We have set $a_{eq} = 10^{-4}$ for matter-radiation equilibrium (while keeping $a_0 = 1$ at present), and our initial condition is $b'(a_{eq}) = 0$. Our results are shown in figures 1 and 2.

We find that modified Chaplygin scenarios start differing from the ΛCDM only recently ($z \simeq 1$) and that, in any case, they yield a density contrast that closely resembles, for any value of $\alpha > 1$, the standard CDM before the present. Notice that ΛCDM corresponds effectively to using Eq. (23) and removing the factor $(1+3(1+w)/\alpha)$ in Eq. (21). Figures 1 and 2 show also that, for any value of α, $b(a)$ saturates as in the ΛCDM case.

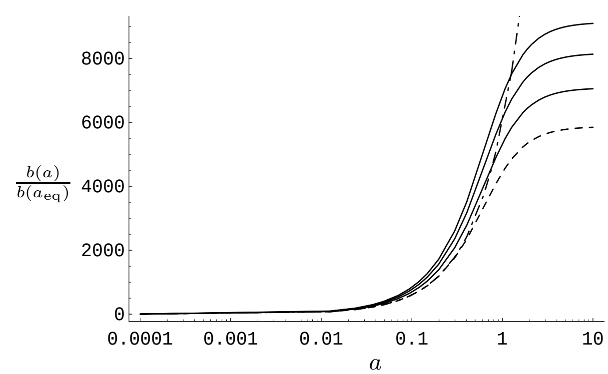

FIG. 2: Evolution of $b(a)/b(a_{\rm eq})$ for the modified Chaplygin gas for $\alpha = 60, 80, 140$ (continuous lines) as compared with ΛCDM (dashed line) and CDM (dashed-dotted line). Lower curves correspond to higher values of α.

In what regards the density contrast, δ, using Eqs. (14), (17) and (25) one can deduce that the ratio between this quantity in the modified Chaplygin and the ΛCDM scenarios is simply given by

$$\frac{\delta_{\rm mChap}}{\delta_{\Lambda\rm CDM}} = \frac{b_{\rm mChap}}{b_{\Lambda\rm CDM}}\frac{\alpha}{\alpha-1} \ , \tag{27}$$

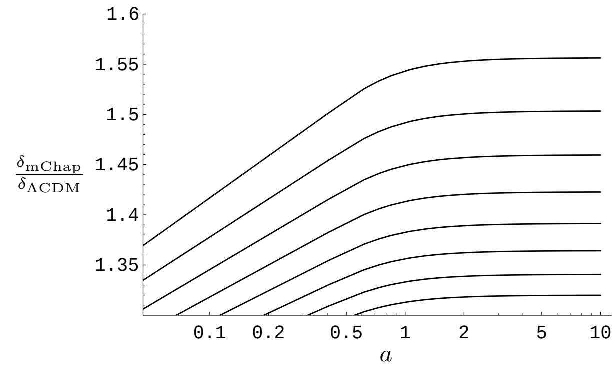

FIG. 3: Evolution of $\delta_{\rm mChap}/\delta_{\Lambda\rm CDM}$ for the modified Chaplygin gas for $\alpha = 60, 65, 70, 75, 80, 85, 90, 95$ (continuous lines) as compared with ΛCDM (dashed line). Lower curves correspond to higher values of α.

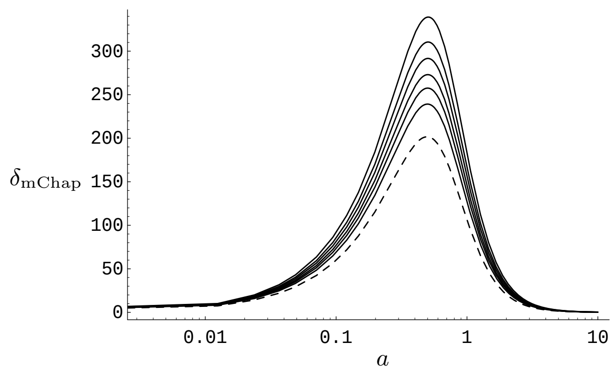

FIG. 4: Evolution of $\delta_{\rm mChap}$ for the modified Chaplygin gas for $\alpha = 50, 60, 70, 85, 105, 150$ (continuous lines) as compared with ΛCDM (dashed line). Lower curves correspond to higher values of α.

and its behavior is depicted in figure 3. We find that it asymptotically evolves to a constant value.

Now, in figure 4, we have plotted δ as a function of a for different values of α. As happens in the the traditional [19, 23] and generalized Chaplygin models, in our models the density contrast decays for large a also.

III. DISCUSSION AND CONCLUSIONS

Using a Zeldovich-like approximation, we have studied the evolution of large-scale perturbations in a recently proposed theoretical framework for the unification of dark matter and dark energy: the so-called modified Chaplygin cosmologies [10], with equation of state

$$p = \frac{1}{\alpha - 1}\left(\rho - \alpha A \rho^{\frac{1}{\alpha}}\right),$$

with $\alpha > 1$. This model evolves from a phase that is initially dominated by non-relativistic matter to a phase that is asymptotically de Sitter. The intermediate regime corresponds to a phase where the effective equation of state is given by $p = 0$ plus a cosmological constant. We have estimated the fate of the inhomogeneities admitted in the model and shown that these evolve consistently with the observations as the density contrast they introduce is smaller than the one typical of CDM scenarios.

On general grounds, the pattern of evolution of perturbations follows is similar to the one in the ΛCDM models and in generalized Chaplygin cosmologies with not very soft equations of state: perturbations in our model either grow or stay at the saturation value. In contrast, in generalized Chaplygin cosmologies with very soft equations of state there seems to be an epoch during which perturbations decrease (although at some stage they stop decreasing and begin to grow again till a saturation value is reached). Therefore, unlike our models, generalized Chaplygin cosmologies models do not always resemble the ΛCDM models, and on these grounds we think ours are more attractive.

As usual, in modified Chaplygin cosmologies, the equation of state parameter w can be expressed in terms of the scale factor and a free parameter α, and the value of the latter can be chosen so that the model resembles as much as desired the ΛCDM model.

Given the short life of this new theoretical setup, no information is available which allows to assert that they should definitely be favored over generalized Chaplygin models, but our results point in that direction. It would be very interesting to move on and compare modified and generalized Chaplygin models from other perspectives, particularly from the observational point of view. We hope this will be addressed in future works.

Acknowledgments

L.P.C. is partially funded by the University of Buenos Aires under project X223, and the Consejo Nacional de Investigaciones Científicas y Técnicas. R.L. is supported by the University of the Basque Country through research grant UPV00172.310-14456/2002 and by the Spanish Ministry of Education and Culture through research grant FIS2004-01626.

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