Resolving the order parameter of High-T$_c$ Superconductors through quantum pumping spectroscopy

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The order parameter of High-T$_c$ superconductors through a series of experiments has been quite conclusively demonstrated to not be of the normal $s-wave$ type. It is either a pure $d_{x^2-y^2}$-wave type or a mixture of a $d_{x^2-y^2}$-wave with a small imaginary $s-wave$ component. In this work a distinction is brought out among the three types, i.e., $s-wave$, $d_{x^2-y^2}-wave$ and $d_{x^2-y^2}+is-wave$ types with the help of quantum pumping spectroscopy. This involves a normal metal double barrier structure in contact with a High-T$_c$ superconductor. The pumped current, heat and noise show different characteristics with change in order parameter revealing quite easily the differences among these.

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I. INTRODUCTION

One of the outstanding issues of High-T$_c$ superconductor research involves the identification of the order parameter symmetry and the underlying mechanism[1,2]. Although a host of experiments have indicated the order parameter symmetry to be of a $d_{x^2-y^2}-wave$ type[3,4] but there are theoretical works[5,6,7,8] which indicate that an imaginary $s-wave$ component is necessary to explain some of the experimental results. These experimental results[9] being notably the splitting of the zero energy peak in conductance spectra which indicates the presence of an imaginary $s-wave$ component which would break the time reversal symmetry. Many theoretical attempts have been made to bring out the differences among the different order parameters. The first theoretical attempts were made by Hu in Ref.[10] where the existence of a sizable areal density of midgap states on the {110} surface of a $d_{x^2-y^2}-wave$ superconductor was brought out. Further using tunneling spectroscopy, Tanaka and Kashiwaya in Ref.[11] brought out the fact that zero bias conductance peaks (which were seen earlier in many experiments[12]) are formed when a normal metal is in contact with a $d_{x^2-y^2}-wave$ superconductor enabling a distinction between $s-wave$ and $d_{x^2-y^2}-wave$ superconductors. A shot noise analysis by Zhu and Ting in Ref.[13] also revealed differences between $s-wave$ and $d_{x^2-y^2}-wave$ superconductors. Further inclusion of phase breaking effects[14] in double barriers formed by normal and superconducting electrodes revealed a double peaked structure in case of $s-wave$ while a dramatic reduction of zero bias maximum for $d_{x^2-y^2}-wave$ superconductors. These are in addition to many other works which involve spin polarized transport in ferromagnet-superconductor junctions[15,16,17] which reveal differences between different possible High-T$_c$ order parameters. In a recent review, Deutscher[18] has used the Andreev-saint James reflections to indicate the presence of an additional imaginary component in the order parameter. Also in another review[19], Lofwander, et. al., arrived at some conclusive arrivals for $d_{x^2-y^2}-wave$ superconductivity in the cuprates. Recently, Ng and Varma[20] studied some of the proposed order parameters and also suggested new experiments to bring out the subtle differences among these. In this work we apply the principles of quantum adiabatic pumping to bring out the differences between the different types of order parameters. Quantum adiabatic pumping involves the transport of particles without the application of any bias voltage. This is done by varying in time atleast two independent parameters of the mesoscopic system out of phase. The physics of the adiabatic quantum pump is based on two independent works by Brouwer in Ref.[21] and by Zhou, et. al., in Ref.[22] which built on earlier works by BPT in Ref.[23]. The first experimental realization of an adiabatic quantum pump was made in Ref.[24]. The phenomenon of quantum adiabatic pumping has been extended to pump a spin current[25] also it has been used in different mesoscopic systems like quantum hall systems[26], luttinger liquid based mesoscopic conductor[27], in the context of quantized charge pumping due to surface acoustic waves[28], a quantum dot in the kondo regime[29], and of course in the context of enhanced pumped currents in hybrid mesoscopic systems involving a superconductor[30,31]. In Ref.[30], Jian Wang, et. al., showed that andreev reflection at the junction between a normal metal and a superconductor (of, $s-wave$ type) can enhance the pumped current as much as four times that in a purely normal metal structure. M. Blauuboer in Ref.[31] showed that for slightly asymmetric coupling to the leads, this enhancement can be slightly increased. Recently, Taddei, et.al. in Ref.[32], generalized the adiabatic quantum pumping mechanism wherein several superconducting leads are present.

This work is organized as follows- After generalizing the formula for the adiabatically pumped current through a normal metal lead in presence of a High-T$_c$ superconductor, we derive the amount of pumped charge current into the normal metal in the vicinity of a High-T$_c$ superconductor with different types of order parameter symmetry. Next we focus on the heat transported and noise generated in the pumping process in case of each of the specific order parameter symmetries. Finally we juxtapose all the obtained results in case of different order parameter symmetry in the amount of pumped current, heat and noise to have some conclusive arrivals and to propose experiments which would fulfill this theoretical proposal.

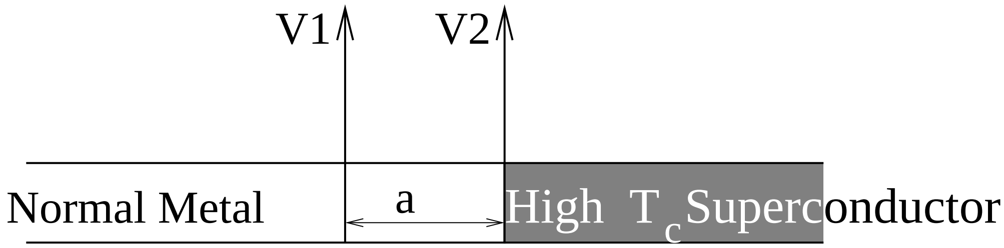

FIG. 1: The model system. A normal metal double barrier structure in proximity with a High T_c superconductor. The double barrier structure is modeled by two delta barriers distance a apart.

II. THEORY OF THE PUMPED CHARGE CURRENT

The model system is shown in Fig. 1. It consists of a normal metal double barrier structure in junction with a High-T$_c$ superconductor. The double barrier structure is modeled by two delta barrier's of strengths V_1 and V_2, a distance 'a' apart. Quantum pumping is enabled by adiabatic modulations in the strength of the delta barriers, i.e., $V_1 = V_0 + V_p sin(wt)$ and $V_2 = V_0 + V_p sin(wt + \phi)$, where V_p is the strength of the pumping amplitude. Andreev reflection mechanism[33,34] is what takes place when a normal metal is brought in contact with a superconductor. The scattering matrix for the entire system is given by:

$$S_{NS}(\epsilon) = \begin{pmatrix} S^{ee}(\epsilon) & S^{eh}(\epsilon) \\ S^{he}(\epsilon) & S^{hh}(\epsilon) \end{pmatrix} \tag{1}$$

wherein $S^{ee}(\epsilon), S^{eh}(\epsilon), S^{he}(\epsilon), S^{hh}(\epsilon)$ are 1X1 matrices, since we are considering single channel leads. The explicit analytical form of the expressions are given by[35]:

$$\begin{aligned}
S^{ee}(\epsilon) &= S_{11}(\epsilon) + \frac{S_{12}(\epsilon)\alpha^h S^*_{22}(-\epsilon)\alpha^e S_{21}(\epsilon)}{1 - \alpha^h\alpha^e S_{22}(\epsilon)S^*_{22}(-\epsilon)}, \\
S^{he}(\epsilon) &= \frac{S^*_{12}(-\epsilon)\alpha^e S^*_{21}(\epsilon)}{1 - \alpha^h\alpha^e S_{22}(\epsilon)S^*_{22}(-\epsilon)}, \\
S^{eh}(\epsilon) &= \frac{S_{12}(\epsilon)\alpha^h S^*_{21}(-\epsilon)}{1 - \alpha^h\alpha^e S_{22}(\epsilon)S^*_{22}(-\epsilon)}, \\
S^{hh}(\epsilon) &= S^*_{11}(-\epsilon) + \frac{S^*_{12}(-\epsilon)\alpha^e S_{22}(\epsilon)\alpha^h S^*_{21}(-\epsilon)}{1 - \alpha^h\alpha^e S_{22}(\epsilon)S^*_{22}(-\epsilon)}. \qquad (2) \\
\text{with, } \alpha^h &= e^{-i\ \arccos[\frac{\epsilon}{\Delta(k_h)}]+i\phi(k_h)}, \alpha^e = e^{-i\ \arccos[\frac{\epsilon}{\Delta(k_e)}]-i\phi(k_e)}, \\
\phi(k_e) &= \ \arccos(\frac{\Delta(k_e)}{|\Delta(k_e)|}), \text{ and } \phi(k_h) = \ \arccos(\frac{\Delta(k_h)}{|\Delta(k_h)|}). \qquad (3)
\end{aligned}$$

where, $\phi(k_e)$ and $\phi(k_h)$ are the phase of the order parameter for electronic like quasiparticles and hole like quasiparticles respectively, with k_e and k_h being the respective wavevectors for the electronic like quasiparticles and hole like quasiparticles[14].

From Refs.[30,31], the adiabatically pumped current into the normal lead in presence of the High-T$_c$ superconducting lead is given by-

$$I = \frac{wq}{2\pi}\int_0^{\tau} d\tau[\frac{dN_L}{dV_1}\frac{dV_1}{dt} + \frac{dN_L}{dV_2}\frac{dV_2}{dt}], \tag{4}$$

The quantity dN_L/dV (wherein, the subscript $_L$ denotes left lead or the normal lead) is the injectivity given at zero temperature by

$$\frac{dN_L}{dV_j} = \frac{1}{2\pi} Im[S^*_{ee}\partial_{V_j} S_{ee} + S^*_{he}\partial_{V_j} S_{he}] \tag{5}$$

and in the weak pumping regime the adiabatically pumped current similar to the analysis in Refs.[21,30], is given by[36]-

$$I = \frac{wqsin(\phi)V_p^2}{\pi} Im[\partial_{V_1} S^*_{ee}\partial_{V_2} S_{ee} + \partial_{V_1} S^*_{he}\partial_{V_2} S_{he}] \tag{6}$$

while for a normal metal structure, the expression for the pumped current in the weak pumping regime is given by-

$$I_N = \frac{wqsin(\phi)V_p^2}{\pi} Im[\partial_{V_1} S^*_{11}\partial_{V_2} S_{11} + \partial_{V_1} S^*_{21}\partial_{V_2} S_{21}] \tag{7}$$

III. PUMPED CURRENT FOR DIFFERENT ORDER PARAMETERS

In Ref.[30], the pumped current for a NS system (where the superconductor is of $s - wave$ type) has been shown to be four times of that in a purely normal metal junction. The system considered in Ref.[30] is also a double barrier delta barrier. We re-derive the results for the pumped current in a normal metal-$s - wave$ superconductor junction and subsequently derive the results for the pumped current in a normal metal-$d_{x^2-y^2}$ wave superconductor junction and for the pumped current in a normal metal - $d_{x^2-y^2} + is$ wave superconducting junction.

$\underline{s - wave\ superconductor}$: For a normal $s - wave$ superconductor which is isotropic $\Delta(k_h) = \Delta(k_e) = \Delta$. If we assume that the Fermi energy is close to the chemical potential of the superconducting lead so that $\epsilon = 0$, then $\alpha^h = \alpha^e = -i$. Further S_{he} is a pure imaginary number $'i'$ for a double barrier structure at resonance, i.e., $S_{he} = -i$, hence $\partial_{V_j} S_{he} = 0$. Thus the second term in Eq. 6 gives zero contribution to the pumped charge current. The first term in Eq. 6 on the other hand gives a non-zero contribution. Similar to the analysis in Ref. [30], for a double barrier pump at resonance, one has $\partial_{V_j} S_{ee} = 2\partial_{V_j} S_{11}$, with $\partial_{V_1} S_{11} = -i/2k$ and $\partial_{V_2} S_{11} = -(i/2k)(S_{12})^2$, for a double barrier quantum dot at resonance $S_{12} = e^{-2ika}$ and thus in the weak pumping regime for an isotropic $s-wave$ superconductor in junction with a normal metal double barrier heterostructure the pumped current denoted by $I(NS)$ is four times that in a pure normal metal structure[30],

$$I(NS) = 4I(N) \text{ with, } I(N) = \frac{wqsin(\phi)V_p^2 sin(4ka)}{4\pi k^2}. \tag{8}$$

$\underline{d_{x^2-y^2} - wave\ superconductor}$: Now we consider the case of a $d_{x^2-y^2} - wave$ superconductor, in junction with a normal metal double barrier structure at resonance. The effective order parameter of the $d_{x^2-y^2} - wave$ superconductor for electron like quasiparticles is $\Delta(k_e) = \Delta_d cos(2\theta_s - 2\alpha)$ and for hole like quasiparticles is $\Delta(k_h) = \Delta_d cos(2\theta_s + 2\alpha)$, with

$$\phi(k_e) = \arccos(\frac{cos(2\theta_s - 2\alpha)}{|cos(2\theta_s - 2\alpha)|}) \text{ and, } \phi(k_h) = \arccos(\frac{cos(2\theta_s + 2\alpha)}{|cos(2\theta_s + 2\alpha)|})$$

In the above equations θ_s is the injection angle between the electron wave vector(k_e) and the x-axis, while α is the misorientation angle between the a axis of the crystal and the interface normal.

Now for a $d_{x^2-y^2} - wave$ superconductor with a (110) orientation, $\alpha = \pi/4$ and we take $\theta_s = \pi/4$, with this $\alpha^h = i$ and $\alpha^e = -i$. Again considering the Fermi energy to be close to the chemical potential of the superconducting lead, so that $\epsilon = 0$, we have for the same double barrier quantum pump at resonance,

$$S_{ee} = S_{11} + (S_{12})^2 S^*_{22}, \text{ and } S_{he} = \alpha^e (S_{12})^2 = -i$$

as because, $|S_{11}|^2 = |S_{22}|^2 = 0$ and $S_{12} = S_{21} = e^{-2ika}$. So we have for the elements in Eq. 6, $\partial_{V_1} S_{he} = \partial_{V_2} S_{he} = 0$, and $\partial_{V_1} S_{ee} = \partial_{V_1} S_{11} + (S_{12})^2 \partial_{V_1} S^*_{22}$. Now from the Dyson equation[37], $\partial_{V_j} G^r_{\alpha\beta} = G^r_{\alpha j} G^r_{j\beta}$, and the fisher-lee relation[38], $S_{\alpha\beta} = -\delta_{\alpha\beta} + i2kG^r_{\alpha\beta}$, one can easily derive $\partial_{V_1} S_{11} = \frac{-i}{2k}$ and $\partial_{V_1} S_{22} = \frac{-i}{2k}(S_{12})^2$. Thus, $\partial_{V_1} S_{ee} = \frac{-i}{2k} + \frac{i}{2k}(|S_{12}|^2)^2$, Now as for a double barrier quantum dot at resonance $|S_{12}|^2 = 1$, we have $\partial_{V_1} S_{ee} = 0$, and hence the pumped current

$I(ND)$ into the normal metal in conjunct with a $d_{x^2-y^2}$-wave superconductor, is zero at resonant transmission through the double barrier structure. Thus,

$$I(ND) = 0 \tag{9}$$

$\underline{d_{x^2-y^2} + is - wave\ superconductor}$: Lastly, we consider the order parameter of the High-T$_c$ superconductor to be of the $d_{x^2-y^2} + is$ type. The $d_{x^2-y^2}$ component has a (110) oriented surface, with $\alpha = \frac{\pi}{4}$ and again we take $\theta_s = \frac{\pi}{4}$. Thus,

$$\Delta(k_e) = \Delta_d cos(2\theta_s - 2\alpha) + i\Delta_s = \Delta_d + i\Delta_s,$$
$$\Delta(k_h) = \Delta_d cos(2\theta_s + 2\alpha) + i\Delta_s = -\Delta_d + i\Delta_s.$$

Now taking, $\Delta_s = z\Delta_d$, we have

$$\phi(k_e) = \arccos(\frac{1+iz}{\sqrt{1+z^2}}), \text{ and } \phi(k_h) = \arccos(\frac{-1+iz}{\sqrt{1+z^2}})$$

If $z = 0.1$, i.e., $s - wave$ part is 10% and $\epsilon = 0$, then from Ref.[39], we have-

$$\begin{aligned}
\alpha^h &= e^{-i \arccos(\frac{\epsilon}{\Delta(k_h)}) + i\phi(k_h)} = 0.548 + 1.095i, \\
\alpha^e &= e^{-i \arccos(\frac{\epsilon}{\Delta(k_e)}) - i\phi(k_e)} = -0.365 - 0.730i \\
\text{with, } \alpha^h\alpha^e &= 0.6 - 0.8i.
\end{aligned}$$

again, by the same arguments as for the case of a $d_{x^2-y^2}$-wave superconductor we have $S_{he} = \alpha^e = -0.365 - 0.73i$ and thus $\partial_{V_j} S_{he} = 0$. Thus the second term of Eq. 6, vanishes for this case also.

Now as for a double barrier structure at resonance, $|S_{22}|^2 = 0$, we have, $S_{ee} = S_{11} + (0.6 - 0.8i)(S_{12})^2 S_{22}^*$, and thus

$$\begin{aligned}
\partial_{V_1} S_{ee} &= \partial_{V_1} S_{11} + (0.6 - 0.8i)(S_{12})^2 \partial_{V_1} S_{22}^*, \\
&= \frac{-i}{2k} + (0.6 - 0.8i)\frac{i}{2k}(|S_{12}|^2)^2 = \frac{0.4}{2k}(2 - i)
\end{aligned}$$

Similarly, $\partial_{V_2} S_{ee} = \frac{-0.8}{2k}(2i + 1)(S_{12})^2$, and for the double barrier structure at resonance $S_{12} = e^{-2ika}$, thus from Eq. 6, the pumped charge current $I(NDs)$ into the normal metal in conjunct with a $d_{x^2-y^2} + is - wave$ superconductor is,

$$\begin{aligned}
I(NDs) &= Im(\partial_{V_1} S_{ee}^* \partial_{V_2} S_{ee}) \\
&= \frac{-0.4wqV_p^2}{\pi k^2} sin(\phi) cos(4ka)
\end{aligned} \tag{10}$$

Thus, the ratio of the pumped current in presence of the High-T$_c$ superconductor to that in a pure normal metal double barrier structure (see Eq. 8) is

$$\frac{I(NDs)}{I(N)} = -1.6cot(4ka), \text{ and for } ka \to n\pi, \frac{I(NDs)}{I(N)} \to \infty. \tag{11}$$

Thus large (infinite) enhancement of the pumped current much more than that in a pure $s - wave$ superconductor is seen for the case of a $d_{x^2-y^2} + is - wave$ superconductor for the specific case when $ka \to n\pi$, with $n = 0, 1, 2....$

To conclude this section we have seen contrasting results in all the three cases, while as seen before for the $s - wave$ case there is four fold enhancement as compared to the normal metal case, in case of a $d_{x^2-y^2} - wave$ superconductor there is no pumped current at all, and lastly for the case of a $d_{x^2-y^2} + is - wave$ superconductor the enhancement can be of infinite magnitude for the special case when $ka \to n\pi$, where $n = 0, 1, 2...$.

IV. PUMPED HEAT AND NOISE

A time dependent scatterer always generates heat flows and can be considered as a mesoscopic (phase coherent) heat source which can be useful for studying various thermoelectric phenomena in mesoscopic structures. The adiabatic quantum pump thus not only generates an electric current but also heat current which can be expressed as the sum of noise power and the joule heat dissipated[40,41,42]. In this section we look into the heat pumped and the noise generated for the various order parameters of the High-T_c superconductors considered above to further unravel the differences among them.

The expressions for pumped heat and noise in the presence of a superconducting ($s-wave$) lead have been earlier derived in Ref.[41]. Below we extend the description to include the $d_{x^2-y^2}-wave$ and $d_{x^2-y^2}+is-wave$ superconductors. The pumped current in Eq. 6, can be re-expressed as follows-

$$I=\frac{wq}{2\pi}\int dE(-\partial_E f)\int_0^\tau dt\sum_{j=1,2}[Im(S^*_{ee}\partial_{V_j}S_{ee}+S^*_{he}\partial_{V_j}S_{he})]\frac{dV_j}{dt} \tag{12}$$

as in the adiabatic regime $\partial_t S_{\alpha\beta}=\sum_i[\partial_{V_i}S_{\alpha\beta}\partial_t X_i+...]$, and from complex algebra $Im[S^*_{ee}\partial_t S_{ee}]=-i[S^*_{ee}\partial_t S_{ee}]$, the pumped current becomes-

$$I=\frac{wq}{2\pi}\int dE\int_0^\tau dt[S^\dagger_{NS}\{f(E+i\frac{\partial_t}{2})-f(E)\}S_{NS}]_{ee} \tag{13}$$

with S_{NS} being the $2X2$ matrix is as defined in Eq. 1, in the above equation the Fermi Dirac distribution is expanded to first order in ∂_t only and $[...]_{ee}$ represents the ee^{th} element of the quantity in brackets.

The heat current pumped is defined as the magnitude of the electric current multiplied by energy measured from the Fermi level.

$$H=\frac{1}{\pi\tau}\int_0^\tau\int dE(E-E_F)[S_{NS}(E,t)f(E+i\frac{\partial_t}{2})-f(E)S^\dagger_{NS}(E,t)]_{ee} \tag{14}$$

Expanding $f(E+i\frac{\partial_t}{2})$ up-to second order one gets a non-vanishing contribution to the heat current in the zero temperature limit as-

$$H=\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee} \tag{15}$$

and since two parameters are being varied, we have

$$H=\frac{1}{8\pi\tau}\int_0^\tau dt\sum_{i,j=1,2}[\partial_{V_i}S^*_{ee}\partial_{V_j}S^*_{ee}+\partial_{V_i}S^*_{he}\partial_{V_j}S^*_{he}]\frac{\partial V_i}{\partial t}\frac{\partial V_j}{\partial t} \tag{16}$$

By integrating the above expression up-to $\tau=2\pi$ we get the pumped current in the weak pumping regime as:

$$H=\frac{w^2}{16\pi}[V_1^2\sum_{\beta=e,h}|\partial_{V_1}S_{\beta e}|^2+V_2^2\sum_{\beta=e,h}|\partial_{V_2}S_{\beta e}|^2+2V_1X_2cos(\phi)\sum_{\beta=e,h}Re(\partial_{V_1}S_{\beta e}\partial_{V_2}S^*_{\beta e})] \tag{17}$$

Similar to the above one can derive expressions for the noise and joule heat dissipated. The expression for the heat current can be re-expressed as -

$$\begin{aligned}H&=\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee}\\&=\frac{1}{8\pi\tau}\int_0^\tau dt\sum_{\beta=e,h}[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{e\beta}[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{\beta e}\end{aligned} \tag{18}$$

The diagonal term is identified as the joule heat while the off-diagonal element is the noise power[41].

$$\begin{aligned} H &= J+N, \\ &= \frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{ee}[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee} \\ &+ \frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{eh}[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{he} \end{aligned} \quad (19)$$

Similar to the analysis for the pumped heat current, the joule heat dissipated and the noise power can be expressed in the weak pumping regime as

$$\begin{aligned} J &= \frac{w^2}{16\pi}[V_1^2\{\sum_{\beta=e,h}|S_{e\beta}\partial_{V_1}S_{e\beta}|^2+2Re(S^*_{ee}S_{eh}\partial_{V_1}S_{ee}\partial_{V_1}S^*_{eh})\}+V_2^2\{\sum_{\beta=e,h}|S_{e\beta}\partial_{V_2}S_{e\beta}|^2+2Re(S^*_{ee}S_{eh}\partial_{V_2}S_{ee}\partial_{V_2}S^*_{eh})\} \\ &+ 2V_1V_2cos(\phi)\{\sum_{\beta=e,h}|S_{e\beta}|^2Re(\partial_{V_1}S_{e\beta}\partial_{V_2}S^*_{e\beta})+Re(S^*_{eh}S_{ee}\partial_{V_1}S_{eh}\partial_{V_2}S^*_{ee})+Re(S^*_{ee}S_{eh}\partial_{V_1}S_{ee}\partial_{V_2}S^*_{eh})\}] \end{aligned} \quad (20)$$

while the noise power is given as below:

$$\begin{aligned} N &= \frac{w^2}{16\pi}[V_1^2\{\sum_{\beta=e,h}|S_{h\beta}\partial_{V_1}S_{e\beta}|^2+2Re(S^*_{he}S_{hh}\partial_{V_1}S_{ee}\partial_{V_1}S^*_{eh})\}+V_2^2\{\sum_{\beta=e,h}|S_{h\beta}\partial_{V_2}S_{e\beta}|^2+2Re(S^*_{he}S_{hh}\partial_{V_2}S_{ee}\partial_{V_2}S^*_{eh})\} \\ &+ 2V_1V_2cos(\phi)\{\sum_{\beta=e,h}|S_{h\beta}|^2Re(\partial_{V_1}S_{e\beta}\partial_{V_2}S^*_{e\beta})+Re(S^*_{hh}S_{he}\partial_{V_1}S_{eh}\partial_{V_2}S^*_{ee})+Re(S^*_{he}S_{hh}\partial_{V_1}S_{ee}\partial_{V_2}S^*_{eh})\}] \end{aligned} \quad (21)$$

Now for our considered system i.e.,a double barrier quantum dot at resonance, we have seen in the previous section that $\partial_{V_j}S_{he}=\partial_{V_j}S_{eh}=0$ regardless of the order parameter symmetry of the High-T$_c$ superconductor and hence the expressions for the pumped heat, noise and joule heat dissipated reduce to-

$$H = \frac{w^2}{16\pi}[V_1^2|\partial_{V_1}S_{ee}|^2+V_2^2|\partial_{V_2}S_{ee}|^2+2V_1V_2cos(\phi)Re(\partial_{V_1}S_{ee}\partial_{V_2}S^*_{ee})] \quad (22)$$

$$J = \frac{w^2}{16\pi}|S_{ee}|^2[V_1^2|\partial_{V_1}S_{ee}|^2+V_2^2|\partial_{V_2}S_{ee}|^2+2V_1V_2cos(\phi)Re(\partial_{V_1}S_{ee}\partial_{V_2}S^*_{ee})] \quad (23)$$

$$N = \frac{w^2}{16\pi}|S_{he}|^2[V_1^2|\partial_{V_1}S_{ee}|^2+V_2^2|\partial_{V_2}S_{ee}|^2+2V_1V_2cos(\phi)Re(\partial_{V_1}S_{ee}\partial_{V_2}S^*_{ee})] \quad (24)$$

Now analyzing the above expressions for the different order parameters, we have-

$\underline{s-wave\ superconductor}$: In the $s-wave$ case as we have already seen $\partial_{V_1}S_{ee}=2\partial_{V_1}S_{11}=-i/k$ and $\partial_{V_2}S_{ee}=2\partial_{V_2}S_{11}=-i/k(S_{12})^2$. With this, the expression for the heat current pumped, joule heat dissipated and noise power reduces to-

$$H = \frac{w^2}{16\pi k^2}[V_1^2+V_2^2-2V_1V_2cos(\phi)cos(4ka)], \quad (25)$$

$$J = \frac{w^2}{16\pi k^2}|S_{ee}|^2[V_1^2+V_2^2-2V_1V_2cos(\phi)cos(4ka)], \quad (26)$$

$$N = \frac{w^2}{16\pi k^2}|S_{he}|^2[V_1^2+V_2^2-2V_1V_2cos(\phi)cos(4ka)]. \quad (27)$$

Thus as is evident from the expression for the pumped noise, the quantum pump is non-optimal[43] (or, non-noiseless), only in the special case when $ka=\phi=n\pi, n=0,1,...$ and with $V_1=V_2$ is the optimality condition met. Of-course, $\phi=n\pi$ implies that in this case there is no charge current as well.

$\underline{d_{x^2-y^2}-wave\ superconductor}$: In this case as also seen earlier, we have $\partial_{V_1}S_{ee}=0$ and $\partial_{V_2}S_{ee}=0$. Thus there is no heat pumped neither any noise generated nor any joule heat dissipated. Thus the pump in conjunct with a $d_{x^2-y^2}$-wave superconductor is cent-percent optimal for any configuration of the parameters and under any condition.

Order Parameter→	$s-wave$	$d_{x^2-y^2}-wave$	$d_{x^2-y^2}+is-wave$
Pumped↓			
Charge	$I(NS)/I(N) \rightarrow 4$	0	$I(NDs)/I(N) \rightarrow \infty$ for $ka \rightarrow n\pi$
Heat	$\propto [V_1^2+V_2^2-2V_1V_2cos(\phi)cos(4ka)]$	0	$\propto [V_1^2+4V_2^2-4V_1V_2cos(\phi)sin(4ka)]$
Noise	*Non-Optimal**	*cent percent Optimal*	*Non-Optimal**

TABLE I: A comparative analysis of pumped charge, heat and noise in cases of $s-wave$, $d_{x^2-y^2}-wave$ and $d_{x^2-y^2}+is-wave$ superconductors in conjunct with a normal metal double barrier structure. [$*$ Optimal under special circumstances (see section Pumped Heat and Noise)].

$\underline{d_{x^2-y^2}+is-wave\ superconductor}$: In this case as also considered earlier the $s-wave$ component is 10% and thus $\partial_{V_1}S_{ee}=(0.4)(2-i)/2k$ while $\partial_{V_2}S_{ee}=-(0.8)(2i+1)e^{-4ika}/2k$, with this the pumped heat, noise and joule heat dissipated in the pumping process reduces to:

$$H = \frac{0.2w^2}{16\pi k^2}[V_1^2+4V_2^2-4V_1V_2cos(\phi)sin(4ka)] \tag{28}$$

$$J = \frac{0.2w^2}{16\pi k^2}|S_{ee}|^2[V_1^2+4V_2^2-4V_1V_2cos(\phi)sin(4ka)] \tag{29}$$

$$N = \frac{0.2w^2}{16\pi k^2}|S_{he}|^2[V_1^2+4V_2^2-4V_1V_2cos(\phi)sin(4ka)] \tag{30}$$

From the expressions it is self evident that the pump is non-optimal but in some special situation it is optimal, i.e., if $\phi=n\pi, n=0,1,...$ and if $ka=n\pi/2, n=1,2,..$ with $V_1=2V_2$ in contradistinction to the s-wave case, one can achieve conditions of optimality.

To end this section we have seen that the pumped heat and noise generated in the pumping process can also show marked differences for the various order parameters considered. In the $s-wave$ and the $d_{x^2-y^2}+is-wave$ cases while the system is non-optimal in the $d_{x^2-y^2}-wave$ case it is cent percent optimal. Further more in both the $s-wave$ and the $d_{x^2-y^2}+is-wave$ cases the pump may be turned optimal in some special situations. These situations on the other hand are different for the two cases, while for the $s-wave$ case the strengths of the delta barriers have to be identical with zero phase difference among them and $ka \rightarrow n\pi, n=0,1,2...$, in the $d_{x^2-y^2}+is-wave$ case the strength of one of the delta barrier's should be twice that of the other again with zero phase difference among them, for the pump to be turned into the optimal regime in case $ka \rightarrow n\pi/2, n=0,1,2....$

V. EXPERIMENTAL REALIZATION

The experimental realization of the above pumping procedure is not difficult, by tuning the Fermi energy of the quantum dot (by applying some suitable gate voltage) one can achieve resonant condition. After this the two delta barrier's can be some external fields, modulating these in time will enable a pumped charge (also heat and noise) current to flow.

VI. CONCLUSIONS

To conclude we have given a simple procedure to distinguish various order parameters proposed in the context of High-T$_c$ superconductivity. In the table 1 above we juxtapose the results obtained in this work. The pumped charge current, heat pumped and noise generated for the three cases considered that of the $s-wave$, $d_{x^2-y^2}-wave$ and $d_{x^2-y^2}+is-wave$ vary markedly which easily reveals the differences among the three.

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