

# Vortex state in a superfluid Fermi gas near a Feshbach resonance

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(Dated: November 1, 2004)

We consider a single vortex in a superfluid Fermi gas in the BCS-BEC crossover regime near a Feshbach resonance. The effect of the molecular Bose condensate upon the vortex structure is discussed within the mean field approximation at zero temperature. Using the self-consistent Bogoliubov-de Gennes equation of the fermion-boson coupled model, we calculate the density distributions of atoms and molecules. As the number of the molecules increases, both atomic and molecular density changes from BCS-like distribution to BEC-like. In this regime, the vortex core size does not show drastic change.

PACS numbers: 03.75.Ss, 03.75.Lm, 67.57.Fg

## I. INTRODUCTION

The evidence of fermionic superfluidity in trapped atomic gases has been clearly shown in the recent experiments [1, 2, 3, 4, 5]. The significant feature of these systems is that the strength of the inter-atomic interaction is manipulated via a Feshbach resonance (FR) phenomena. Therefore the interaction dependence of Fermi gas can be studied. The crossover, which has been discussed several decades [6, 7], between weak coupling Bardeen-Cooper-Schrieffer (BCS) superfluid and Bose Einstein condensation (BEC) of pre-formed pairs can be realized in these systems.

The FR phenomena involve the scattering of atoms from open channel states into a molecular bound state formed from neighboring closed channel states. The so-called “pre-formed pair” in the BCS-BEC crossover theory is here equivalent to the substantial molecule in the closed channel, which has been observed as a BEC on the one side of the FR.

In this paper we discuss the property of a single vortex in the BCS-BEC crossover regime at zero temperature. Since the vortex states in BCS superfluids and in BECs are significantly differ in their distributions of particles, observing a vortex state in a Fermi gas provides the information about superfluidity in the crossover regime. In the case of a vortex in the BCS state, the energy gap is suppressed at the vortex core and, therefore, the particle-hole symmetrical modes, which are localized at the vortex core, exist in the vicinity of the Fermi surface [8]. In contrast to the pair amplitude, the atomic density is finite at the center of the vortex core. On the other hand, the particle density in a BEC vanishes at the vortex core, directly reflecting a singularity of the order parameter.

We are also interested in the vortex core size in the crossover regime. The core size of BCS superfluid is given by  $\xi_{\text{BCS}} \sim \hbar v_F / \Delta$ , where  $v_F$  is the Fermi velocity and  $\Delta$  is the energy gap. As the attractive interaction becomes strong, the growth of the energy gap makes the

core size smaller [9]. As for a molecular BEC, the core size is written by  $\xi_{\text{BEC}} \sim \sqrt{1/8\pi n_M a_M}$ , where  $n_M$  is the number density of molecules and  $a_M$  is the  $s$ -wave scattering length of a molecule, which diverges at resonance as well as that of an atom [10, 11]. So  $\xi_{\text{BEC}}$  also becomes small near resonance. Then, does the core size have its minimum around the resonance?

There are several theoretical papers concerning vortex states in Fermi gases [9, 12, 13, 14, 15, 16]. Bulgac and Yu, using their superfluid local density approximation, found that in the BCS regime vortices give rise to a depletion in density [15]. Tempere *et. al.* employed a path-integral method and found the smooth change of the core size in the crossover regime [16]. These papers, however, deal with only Fermi gases of which interaction strength is manipulated, and the effect of molecular BEC is not considered. In this paper, we start with an atom-molecule coupled model [17, 18, 19], and calculate density distributions and order parameters of both atoms and molecules. We use the mean field approximation to simply deal with the effect of molecular BEC.

## II. FORMALISM

We consider atomic gases of two atomic hyperfine states (labeled by  $\sigma = \uparrow, \downarrow$ ), which are coupled to a molecular two-particle bound state. The energy of a bare molecule relative to that of two bare atoms is denoted by  $2\nu$ . The model Hamiltonian of the fermion-boson coupled system is given by

$$\begin{aligned} \hat{H} = \int d\mathbf{r} & \left[ \sum_{\sigma} \hat{\psi}_{\sigma}^{\dagger}(\mathbf{r}) \mathcal{H}_A(\mathbf{r}) \hat{\psi}_{\sigma}(\mathbf{r}) \right. \\ & + \hat{\phi}^{\dagger}(\mathbf{r}) \{ \mathcal{H}_M(\mathbf{r}) + 2\nu \} \hat{\phi}(\mathbf{r}) \\ & \left. + g \{ \hat{\phi}^{\dagger}(\mathbf{r}) \hat{\psi}_{\downarrow}(\mathbf{r}) \hat{\psi}_{\uparrow}(\mathbf{r}) + \text{h.c.} \} \right], \quad (1) \end{aligned}$$

where  $\hat{\psi}_{\sigma}$  and  $\hat{\phi}$  are the field operators of atoms and molecules, respectively. The first and second terms in the integration represent the bare Hamiltonian of atoms and molecules, respectively, where  $\mathcal{H}_A(\mathbf{r}) = -(\hbar^2/2m)\nabla^2 +$

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$V_A(\mathbf{r})$ ,  $\mathcal{H}_M(\mathbf{r}) = -(\hbar^2/4m)\nabla^2 + V_M(\mathbf{r})$ ,  $m$  is the atomic mass and  $V_{A,M}(\mathbf{r})$  are the trapping potential for atoms and molecules, respectively. The last term of Eq. (1) represents the atom-molecule coupling associated with FR. Its contribution to the effective atom-atom interaction is  $-g^2/2\nu$ . This means that the interaction is manipulated by changing  $\nu$ . So we investigate the  $\nu$  dependence of this system, especially around  $\nu \sim 0$ . Although there exist non-resonant inter-atomic and intermolecular interactions, we assume the effect of the resonant process is so dominant that other processes are negligible.

To conserve the total number of particles, we use a single chemical potential  $\mu$  and work with  $\mathcal{H} = \hat{H} - \mu\hat{N}$ , where  $\hat{N}$  is the number operator given by

$$\hat{N} = \int d\mathbf{r} \left[ \sum_{\sigma} \hat{\psi}_{\sigma}^{\dagger}(\mathbf{r})\hat{\psi}_{\sigma}(\mathbf{r}) + 2\hat{\phi}^{\dagger}(\mathbf{r})\hat{\phi}(\mathbf{r}) \right]. \quad (2)$$

Here, we introduce the mean-field order parameters  $\mathcal{P}(\mathbf{r}) \equiv \langle \hat{\psi}_{\uparrow}(\mathbf{r})\hat{\psi}_{\uparrow}(\mathbf{r}) \rangle$  and  $\phi(\mathbf{r}) \equiv \langle \hat{\phi}(\mathbf{r}) \rangle$ , which correspond to the Cooper pair amplitude and molecular order parameter, respectively. The number density of molecules are given by  $n_M(\mathbf{r}) = |\phi(\mathbf{r})|^2$ , as usual. Though we neglect a bare atom-atom interaction, the effective interaction via FR supports the pairing of atoms. Since  $\phi(\mathbf{r})$  is related to an energy gap as we will show below, the pair amplitude remains finite as long as a molecular BEC exists. The relation between order parameters is given by the equilibrium condition  $i\hbar\langle \partial\hat{\phi}(\mathbf{r})/\partial t \rangle = \langle [\hat{\phi}(\mathbf{r}), \mathcal{H}] \rangle = 0$ , from which it follows that

$$[\mathcal{H}_M(\mathbf{r}) + 2\nu - 2\mu]\phi(\mathbf{r}) + g\mathcal{P}(\mathbf{r}) = 0. \quad (3)$$

As in the case of the well-known mean field BCS theory, the ‘‘off-diagonal energy’’ should be defined as the energy gap, i.e., Eq. (1) implies that  $\Delta_{\text{eff}}(\mathbf{r}) \equiv g\mathcal{P}(\mathbf{r})$ . Then, conventional Bogoliubov-de Gennes (BdG) approach is adopted in a similar manner: the field operator of the atom is transformed with

$$\begin{bmatrix} \hat{\psi}_{\uparrow}(\mathbf{r}) \\ \hat{\psi}_{\downarrow}(\mathbf{r}) \end{bmatrix} = \sum_j \begin{bmatrix} u_j^*(\mathbf{r}) & -v_j(\mathbf{r}) \\ v_j^*(\mathbf{r}) & u_j(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \alpha_{j\uparrow} \\ \alpha_{j\downarrow} \end{bmatrix}, \quad (4)$$

and the BdG equation is obtained by

$$\begin{bmatrix} \mathcal{H}_A(\mathbf{r}) - \mu & g\phi^*(\mathbf{r}) \\ g\phi(\mathbf{r}) & -\mathcal{H}_A(\mathbf{r}) + \mu \end{bmatrix} \begin{bmatrix} u_j(\mathbf{r}) \\ v_j(\mathbf{r}) \end{bmatrix} = \epsilon_j \begin{bmatrix} u_j(\mathbf{r}) \\ v_j(\mathbf{r}) \end{bmatrix}. \quad (5)$$

In this notation, the pair amplitude and the density at zero temperature are written by  $\mathcal{P}(\mathbf{r}) = -\sum_j u_j^*(\mathbf{r})v_j(\mathbf{r})$  and  $n_A(\mathbf{r}) = 2\sum_j |v_j(\mathbf{r})|^2$ , respectively. To avoid the ultraviolet divergence, we introduce a cutoff energy  $\omega_c$  which is in the order of the Fermi energy, and redefine  $\mathcal{P}(\mathbf{r}) = -\sum_j u_j^*(\mathbf{r})v_j(\mathbf{r})e^{-\epsilon_j^2/\omega_c^2}$ .

Then, all we have to do is to solve Eqs. (3) and (5) self-consistently. The chemical potential is determined so that the total number of atoms  $N_{\text{tot}} = N_A + 2N_M$  is conserved, where  $N_A \equiv \int d\mathbf{r}n_A(\mathbf{r})$  and  $N_M \equiv \int d\mathbf{r}n_M(\mathbf{r})$ .

### III. VORTEX STATE IN THE CROSSOVER REGIME

We assume a two-dimensional optical trap and approximate with a harmonic potential:  $V_A(\mathbf{r}) = (1/2)m\omega^2(x^2 + y^2)$  and  $V_M(\mathbf{r}) = 2V_A(\mathbf{r})$ . For simplicity, we neglect the  $z$  dependence of the order parameters. When considering the microscopic structure of atomic clouds, however, the degree of freedom in  $z$  direction is important. So we assume that gases are confined in the length  $L_z$ , and impose the periodic boundary condition. This approximation is valid when  $L_z$  is much larger than the radial size. A single particle state of a bare atom in this potential is given by  $e^{ikz}e^{il\theta}(r/a_{\text{HO}})^l \mathcal{L}_n^{|l|}(r^2/2a_{\text{HO}}^2)e^{-r^2/4a_{\text{HO}}^2}$  and its energy eigenvalue is  $\epsilon_{klm} = \hbar\omega(2n + |l| + 1 + k^2a_{\text{HO}}^2)$ , where  $(r, \theta, z)$  is the cylindrical coordinate,  $a_{\text{HO}} = \sqrt{\hbar/2m\omega}$  is the harmonic oscillator length, and  $\mathcal{L}_n^l(x)$  is the generalized Laguerre polynomial function. The indices specifying the energy are given by  $l = 0, \pm 1, \pm 2, \dots$ ,  $n = 0, 1, 2, \dots$ , and  $ka_{\text{HO}} = (2\pi a_{\text{HO}}/L_z)n_k \equiv k_0n_k$  where  $n_k = 0, \pm 1, \pm 2, \dots$ . By counting the number of the eigenstates below  $\epsilon_F$ , the Fermi energy as a function of the number of atoms is given by  $\epsilon_F(N_A) = \hbar\omega(15k_0N_A/16)^{2/5}$ . The characteristic energy of the system is defined as  $E_F \equiv \epsilon_F(N_{\text{tot}})$ , which is the Fermi energy in the BCS limit.

For a vortex state in equilibrium, we assume the form

$$\phi(\mathbf{r}) = e^{i\theta}\phi(r), \quad (6)$$

$$u_{klm}(\mathbf{r}) = e^{ikz}e^{i(l-1)\theta}u_{klm}(r), \quad (7)$$

$$v_{klm}(\mathbf{r}) = e^{ikz}e^{i\theta}v_{klm}(r), \quad (8)$$

where  $u_{klm}(r), v_{klm}(r)$  and  $\phi(r)$  are real functions, and  $n = 0, 1, 2, \dots$  is the radial quantum number. In the BEC limit, by substituting  $g\mathcal{P} = 0$  in Eq. (3) the lowest energy state of molecular BEC with a single vortex is given by  $\phi(\mathbf{r}) = \sqrt{N_M/2\pi a_{\text{HO}}^2 L_z} e^{i\theta} f_0(r/a_{\text{HO}})$ , where  $f_0(r) \equiv \sqrt{2}re^{-r^2/2}$ . Here, the chemical potential is also determined by Eq. (3) as  $\mu = \nu + \hbar\omega$  which is composed of the binding energy  $\nu$ , the trapping energy  $\hbar\omega/2$ , and the vortex energy  $\hbar\omega/2$ .

We consider 300 atoms of  ${}^6\text{Li}$  in a trap with  $\omega = 2\pi \times 300$  Hz and  $L_z = 10 \mu\text{m}$ , from which it follows that  $E_F = 9.7\hbar\omega$ ,  $a_{\text{HO}} = 1.7 \mu\text{m}$ , and  $L_z/a_{\text{HO}} = 6$ . As for the coupling constant, we have calculated with  $g\sqrt{n} = E_F$  and  $3E_F$ , where  $n$  is the mean density of total atoms:  $n = N_{\text{tot}}/2\pi a_{\text{HO}}^2 L_z$ . Figure 1 shows the  $\nu$  dependences of (a) the chemical potential and (b) the numbers of atoms and molecules. Since bare atom-atom and molecule-molecule interactions are neglected in the model Hamiltonian (1), the trapped gas behaves as an ideal Fermi gas and Bose gas far from the resonance in each side, i.e., in the BCS limit  $\mu$  goes to  $E_F$ , while  $\mu \rightarrow \nu + \hbar\omega$  in the BEC limit as shown in Fig. 1(a). By comparing the results with  $g\sqrt{n} = E_F$  and  $3E_F$  in Fig. 1, the increase of  $g$  results in mainly two effects: (i) the resonance region in the parameter space of  $\nu$  becomes wide,

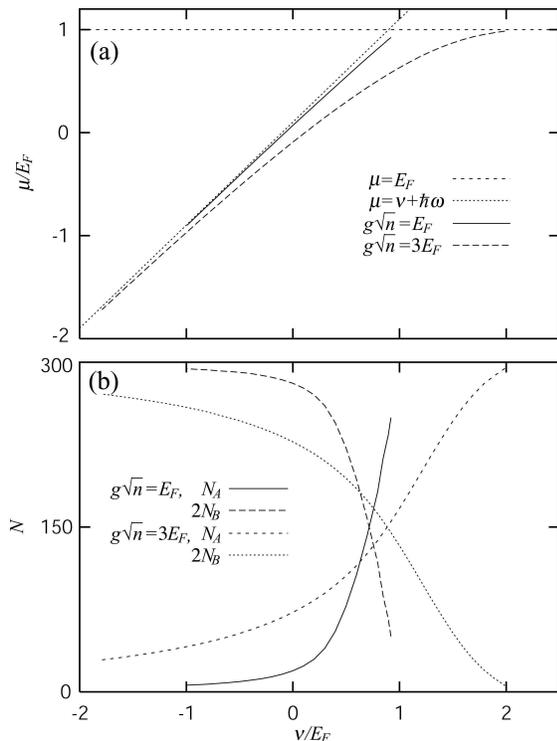


FIG. 1: The  $\nu$  dependence of (a) the chemical potential, and (b) the numbers of atoms and molecules. In both figures, the results with  $g\sqrt{n} = E_F$  and  $3E_F$  are plotted. (a) In both results,  $\mu \rightarrow E_F$  in the BCS limit ( $\nu > 0$ ) and  $\mu \rightarrow \mu + \hbar\omega$  in BEC limit ( $\nu < 0$ ). The reduction of the chemical potential with  $g\sqrt{n} = 3E_F$  is larger than that with  $g\sqrt{n} = E_F$ . (b) The total density is always conserved as  $N_A + 2N_M = 300$ . The crossover region clearly becomes wide as  $g$  increases.

and (ii) the effective energy gap  $g\phi$  increases, leading to the reduction the chemical potential. Therefore, though in the experimental situation the coupling constant is estimated as large as  $g\sqrt{n} \sim 10E_F$ , the calculation with  $g\sqrt{n} = 3E_F$  may bring us qualitative insight into the vortex state.

#### IV. DISCUSSION

Figure 2 shows the density distributions of (a) atoms and (b) molecules for  $\nu/E_F = \{1.9, 1.0, 0.0, -1.0\}$  and  $g\sqrt{n} = 3E_F$ . Both profiles are normalized by the corresponding numbers of each particles. We also plot the density distribution of a non-interacting BEC,  $|f_0(r)|^2$ , in both figures. As we mentioned above, the atomic distribution far from resonance in BCS side ( $\nu/E_F = 1.9$ ) has no explicit hole associated with a vortex. The large distribution in radial direction is also the feature of fermionic profile. As  $\nu$  decreases, however, these fermionic features disappear and the atomic distribution becomes BEC-like, i.e., the cloud becomes small in radial direction and, moreover, the hole at the vortex core clearly appears.

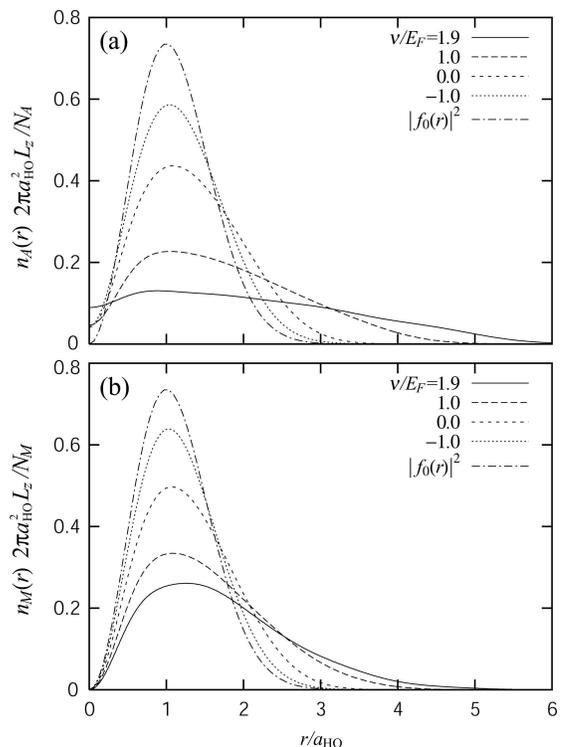


FIG. 2: The density distributions of (a) atoms and (b) molecules at  $g\sqrt{n} = 3E_F$ . Density profile of an ideal Bose gas is also plotted in both figures. Both atoms and molecules change from broad distribution at  $\nu/E_F = 1.9$  to narrow one at  $\nu/E_F = -1.0$ . The fraction of atoms localized at the core decreases as  $\nu$  decreases, while the molecular density at a vortex core is always zero.

It means that the density depletion argued by Bulgac and Yu [15] does not merely come from the increase of  $N_M$ , but also come from the change of the atomic density profile.

The existence of a molecular BEC leads to these changes in atomic distribution. Since the molecular density always vanishes at vortex core as shown in Fig. 2(b), the atomic density is also suppressed there. On the other hand, molecules are also strongly affected in the BCS side, and have a broader distribution than those in BEC side. Coupled to each other, both atoms and molecules change from BCS-like to BEC-like distribution.

Next, we discuss the change of order parameters in the crossover regime. Figure 3 shows  $\nu$  dependence of (a) the pair amplitude  $\mathcal{P}(r)$  and (b) the molecular order parameter  $\phi(r)$ , or the effective energy gap. These are also results of calculation with  $\nu/E_F = \{1.9, 1.0, 0.0, -1.0\}$  and  $g\sqrt{n} = 3E_F$ . As  $\nu$  decreases, the molecular order parameter monotonically grows and becomes narrower as in the number density distribution. On the other hand, the fermionic pair amplitude reaches its maximum at  $\nu \sim E_F$ . As  $\nu$  decreases, first, the pair amplitude increases since the effective energy gap increases, i.e.,  $\phi$  increases. When the energy gap exceeds the chemical po-

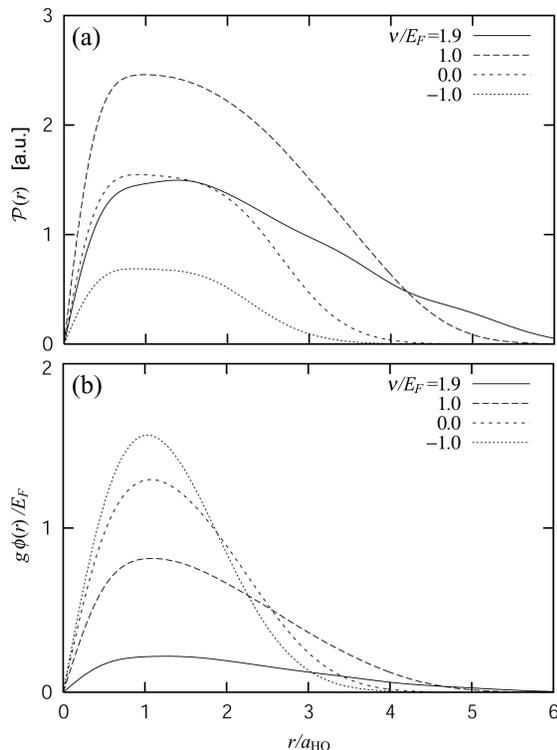


FIG. 3: The  $\nu$  dependence of the mean field order parameters. (a) The pair amplitude of atoms has its maximum at  $\nu/E_F \sim 1.0$ , while (b) The molecular order parameter monotonically grows as  $\nu$  increases.

tential, the pair amplitude saturate. At this point all of atoms take part in the pair amplitude. Then, the pair amplitude decreases as the number of atoms decreases.

In Figs. 3(a) and (b), the order parameters have maxima at the nearly same radius. This means that the core size does not strongly depend on  $\nu$ . Though the extrapolation of BCS and BEC theories predicts that the core size becomes small at resonance, as we mentioned

in Sec. I, our calculation does not show such behavior. This result is consistent with the results of Tempere *et al.* [16]. In our calculation, the trap potential determines the core size as  $a_{\text{HO}}$ , since we neglect bare atom-atom and molecule-molecule interactions. When these interactions are considered, the typical core size may change and depend on  $\nu$  so that the core size gradually change between  $\xi_{\text{BCS}}$  and  $\xi_{\text{BEC}}$ , which are determined by the non-resonant interactions. These interaction terms, however, do not affect the fact that the core size does not become small near resonance. The atom-molecule coupling, which causes the divergence of the scattering length, determines the properties of vortices near resonance.

## V. CONCLUSION

We have studied a vortex state in a superfluid Fermi gas near a Feshbach resonance, using an atom-molecule coupled model and the mean field approximation. Coupled to each other, both atomic and molecular density distributions change from BCS-like to BEC-like. The depletion of the total particle density does not merely come from the increase of the number of molecules. Affected by the molecular BEC the hole at the vortex core in atomic density clearly appears too. We also studied the change of a vortex core size and found that it does not become small even in the strong coupling regime near resonance.

## Acknowledgments

The computation in this work has been done using the facilities of the Supercomputer Center, Institute for Solid State Physics, University of Tokyo. YK would like to acknowledge support from the Fellowship Program of the Japan Society for Promotion of Science (Project No. 16-0648).

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- [1] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
  - [2] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).
  - [3] J. Kinast, S. L. Hemmer, M. E. Gehm, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. **92**, 150402 (2004).
  - [4] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. H. Denschlag, and R. Grimm, Phys. Rev. Lett. **92**, 203201 (2004).
  - [5] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. H. Denschlag, and R. Grimm, Science **305**, 1128 (2004).
  - [6] A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter*, ed. A. Pekalski *et al.* (Springer-Verlag, Berlin, 1980), p. 13.
  - [7] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
  - [8] P. G. de Gennes, *Superconductivity of metals and alloys* (Addison-Wesley, New York, 1989), chap. 5.
  - [9] N. Nygaard, G. M. Bruun, C. W. Clark, and D. L. Feder, Phys. Rev. Lett. **90**, 210402 (2003).
  - [10] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Phys. Rev. Lett. **93**, 090404 (2004).
  - [11] M. Randeria, in *Bose-Einstein Condensation*, ed. A. Griffin *et al.* (Cambridge University Press, New York, 1995), p. 335.
  - [12] M. Rodriguez, G.-S. Paraoanu, and P. Törmä, Phys. Rev. Lett. **87**, 100402 (2001).
  - [13] G. M. Bruun and L. Viverit, Phys. Rev. A **64**, 063606 (2001).
  - [14] N. Nygaard, G. M. Bruun, B. I. Schneider, C. W. Clark, and D. L. Feder, Phys. Rev. A **69**, 053622 (2004).
  - [15] A. Bulgac and Y. Yu, cond-mat/0406256 (2004).

- [16] J. Tempere, M. Wouters, and J. T. Devreese, cond-mat/0410252 (2004).
- [17] E. Timmermans, K. Furuya, P. W. Milonni, and A. K. Kerman, Phys. Lett. A **285**, 228 (2001).
- [18] M. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Phys. Rev. Lett. **87**, 120406 (2001).
- [19] Y. Ohashi and A. Griffin, Phys. Rev. Lett. **89**, 130402 (2002).