

Monte Carlo simulation of Ising model on directed Barabasi-Albert network

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Abstract:

The existence of spontaneous magnetization of Ising spins on directed Barabasi-Albert networks is investigated with seven neighbors, by using Monte Carlo simulations. In large systems we see the magnetization for different temperatures T to decay after a characteristic time $\tau(T)$, which is extrapolated to diverge at zero temperature.

Keywords: Monte Carlo simulations, Directed Barabasi-Albert networks, Magnetization, Fortran program

Introduction:

The Ising magnet is since decades a standard tool of computational physics [1]. We apply it here to scale-free networks [2], where previous simulations [3] indicated a Curie temperature increasing logarithmically with increasing system size N . In contrast to that work we use here directed [4] as opposed to undirected networks and then apply the standard Glauber kinetic Ising model [1] to the fixed network.

Directed Barabasi-Albert network:

Putting Ising spins onto the sites (vertices, nodes) of a network, we simulate our Ising magnetic model on directed Barabasi-Albert networks. The Barabasi-Albert network is grown such that the probability of a new site to be connected to one of the already existing sites is proportional to the number of previous connections to this already existing site: The rich get richer. In this way each new site selects exactly m old sites as neighbours.

Then each spin is influenced by the fixed number m of neighbours which it had selected when joining the network. It is not influenced by other spins which selected it as neighbour after it joined the network.

The Barabasi-Albert network is simulated by a Fortran program calculating the neighbours:

```

    parameter( nsites=500000,m=7,iseed=3, maxmax=20000,
1 max=nsites+m, length=1+2*m*nsites+2*m*m , T=1.0)
    integer*8 ibm, iex
    dimension list(length), is(max), iex(2*m+1), neighb(max,m)
    ibm=iseed-1
    factor=(0.25d0/2147483648.0d0)/21474836484.0d0
    do 7 i=1,m
        do 7 nn=1,m
            neighb(i,nn)=nn
7         list((i-1)*m+nn)=nn
        L=m*m
c     All m initial sites are connected
        do 1 i=m+1,max
            do 2 new=1,m
4             ibm=ibm*16807
                j=1+(ibm*factor+0.5)*L
                if(j.le.0.or.j.gt.L) goto 4
                j=list(j)
                list(L+new)=j
                list(L+m+new)=i
2             neighb(i,new)=j
1         L=L+2*m
c     print *,ibm,neigh
c     end of network and neighbourhood construction

```

At each step, a new spin is added which builds m new connections `neighb`, randomly to already existing spins. The probability for an existing spin to be chosen as neighbour is proportional to the number of its neighbours, with the help of the Kertesz `list`.

Ising Magnet using Monte Carlo Simulations:

First we initialize a directed Barabasi-Albert network with m neighbours (all m initial spins are connected with each other and themselves), here $m = 7$. We put an Ising spins onto every site, with all spins up, because we test here for ferromagnetism. Then with the standard Glauber (heat bath) Monte Carlo algorithm spins we search for thermal equilibrium at positive temperature. (All temperatures are given in units of coupling constant over Boltzmann constant and change between 0.5 and 1.0 only).

After putting all spins on the network, we go through the whole network and use the Monte Carlo step (MCS) on every spin; we say that we make one MCS per spin at each time step. Each spin is influenced by its exactly m neighbours. We calculate the magnetization versus the number of time steps, with the same number of neighbours m and different temperatures T .

Initially we start with $T = 1.0$, and a number of spins equal to 500,000, and time up to 20,000. Then we change the temperatures from 1.0 to lower values, for three samples with three `iseed` random numbers.

So we can draw a graph of magnetization versus time for different temperatures to see how the magnetization changes, Fig. 1.

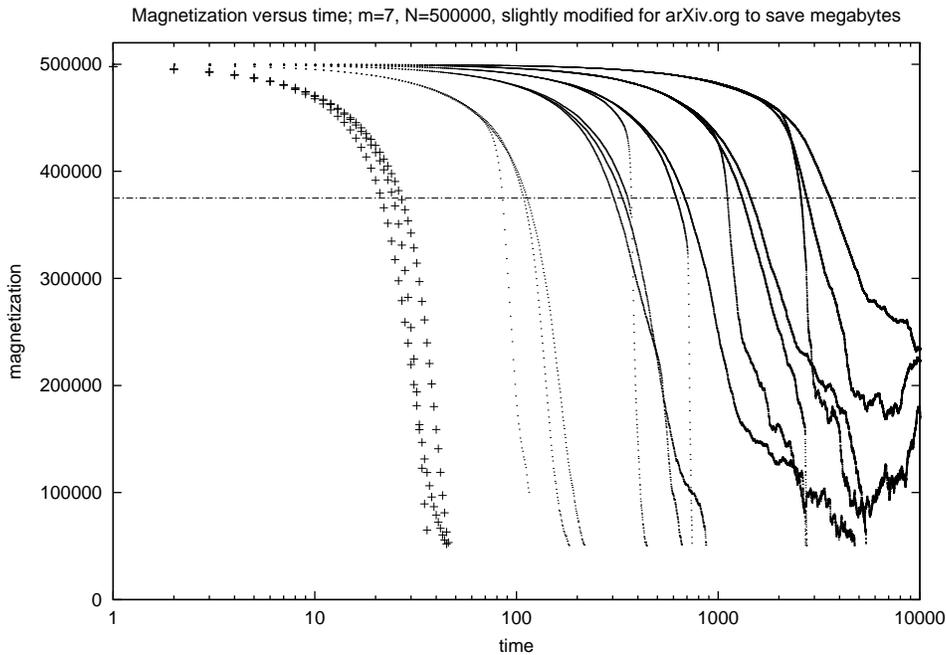


Figure 1: Magnetization versus MCS per spin, for $N = 500000$, time up to 20000, $m = 7$, `iseed` = 1,3,5 for different temperatures.

Now we compare these graphs with each other by two ways:

1. We determine the time τ_1 after which the magnetization has decayed to 3/4 of its initial value (here 375,000). This is done by plotting all results of different temperatures in one graph, we can draw the horizontal line “magnetization equal 375,000”; then the nearly vertical lines cross the horizontal

line at the time τ_1 . So we get different values of τ_1 for different temperatures. Then we draw the graph $1/\log_{10} \tau_1$ versus temperature as seen in Fig. 2.

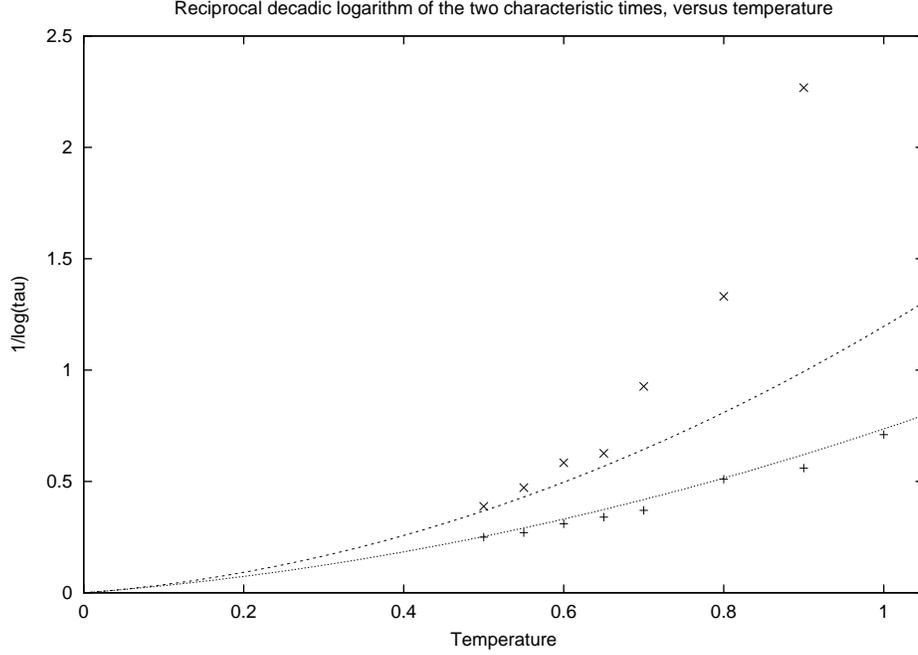


Figure 2: $1/\log_{10}(\tau)$ versus temperature for $N = 500000$, time up to 20000, $m = 7$, $\text{iseed} = 1,3,5$, three sample. The + signs refer to τ_1 , the x signs to τ_2 . The curves are parabolas corresponding to an asymptotic Arrhenius law $\tau \propto \exp(8.3/T)$.

2. Alternatively we define a $\tau_2(T)$ such that the magnetization curves $M(t, T)$, plotted as a function of the scaled time $t/\tau_2(T)$, agree with those for the reference temperature $T = 1.0$ where $\tau_2 = 1$. Thus we get values of τ_2 of each sample, changing with different temperatures. Then we take the decadic logarithm of the average τ_2 to draw $1/\log(\tau_2)$ versus temperature as shown also in Fig. 2.

Conclusion:

We see that all figures agree with the modified Arrhenius law:

$$1/\ln(\tau) = 0.12 \cdot T + \text{const} \cdot T^2$$

meaning that for each positive temperature there is a finite relaxation time after which the initial magnetization decays towards zero: Similar to the one-dimensional Ising model there is no ferromagnetism on this directed Barabasi-Albert network.

References:

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