BI-TP 2004/32

On the conditions driving the chemical freeze-out

A. Tawfik*

University of Bielefeld, P.O. Box 100131, D-33501 Bielefeld, Germany

(Dated: 31st October 2004)

Abstract

We propose the entropy density as the thermodynamic condition driving the chemical freeze-out. Taking its value from lattice calculations for two and three quark flavors, we find that it is excellent in reproducing the experimentally estimated freeze-out parameters. The two characteristic endpoints of the freeze-out curve are reproduced as well.

*Electronic address: tawfik@physik.uni-bielefeld.de

I. INTRODUCTION

At low temperatures T, matter consists of confined hadrons. At the critical temperature T_c, it is conjectured that the hadrons are dissolved into quark-gluon plasma (QGP). Reducing the temperature of QGP leads to hadronization, the transition from deconfined QGP to confined hadrons. At a certain temperature T_{ch}, the system goes into chemical equilibration, freezing-out. Below this value, thermal equilibration takes place and the compositions of matter turn once again to be the non-interacting confined hadrons.

Theoretical characterize of the freeze-out curve is still lacking. We assume that the system at T_{ch} still has inelastic interactions but in chemical equilibration, i.e. there is a balance between particle-absorption and -production. The best experimental way to determine the freeze-out parameters T_{ch} and μ in high-energy collisions is to measure the ratios of particle yields. It has been found that the thermal models are very well able to reproduce the particle ratios at different incident energies [1, 2, 3]. The question we intent to answer is: what is the universal thermodynamic condition which describes the freeze-out curve at all incident energies?

To answer this question, we first recall the physical chemistry. *Without energy input the chemical reactions always proceed toward equilibrium.* The equilibrium constant $\mathcal{C}_{eq}$ in the reaction $aA + bB \leftrightarrow cC + dD$ can be calculated according to the "law of mass action" [4]

$$\mathcal{C}_{eq} = \frac{[C]_{eq}^{c} \ [D]_{eq}^{d}}{[A]_{eq}^{a} \ [B]_{eq}^{b}}, \tag{1}$$

where (A, B) and (C, D) are the reactants and products, respectively. (a, b) and (c, d) are the corresponding concentrations. In heavy-ion collisions, the decay channels, from which the different particles are produced, looks alike this simple chemical reaction. The backward direction can be viewed as an annihilation or absorption process. The change in Gibbs free energy determines the likely direction. When the concentrations in Eq. (1) are known, then for an ideal gas (no enthalpy change),

$$\delta G \approx \delta G^0 + T \ln \mathcal{C}_{eq}. \tag{2}$$

The standard free energy δG^0 is given by $\mathcal{C}_{eq}$ in Eq. (1) at $T \neq T_{ch}$. At equilibrium, δG vanishes and therefore $\mathcal{C}_{eq} \approx \exp(-\delta G^0/T)$

$$s = \ln \left(1/\mathcal{C}_{eq}\right) - n \left(\frac{\mu}{T}\right), \tag{3}$$

where s and n are the equilibrium entropy and particle density, respectively. At $\mu = 0$, $s = \ln(1/\mathcal{C}_{eq})$. With increasing μ, s monotonically decreases.

Second, we recall phenomenological observation: With increasing the incident energy, an increase in the particle production is expected. The hadron resonance gas model (HRGM) is based on this observation. The chemical potential μ relates the energy change with the particle number. This relation is controlled by "second law of thermodynamics", the entropy. At equilibrium, the entropy gives the amount of energy that produces no further work in the system.

The primary tool for measuring the chemical equilibrium at a certain incident energy is to measure the multiplicities of different produced particles. The statistical models are used to fit the experimental particle ratios. The resulting fit parameters are T_{ch} and μ. Our objective in this work is to define one universal condition to describe these parameters at different energies.

Apparently, the freeze-out curve has two characteristic points: one at $T_{ch} \neq 0$ and $\mu = 0$ and another at $T_{ch} = 0$ and $\mu \neq 0$. Any suggested model has to be able to reproduce both of them simultaneously. The first point has been the subject of different experimental studies [5]. It has been found that $T_{ch}(\mu = 0) \approx 174$ MeV. From lattice simulations for different quark flavors, the resulting transition temperature has almost the same value as $T_{ch}(\mu = 0)$. This implies that the transition and freeze-out lines seem to be coincident at low μ. For the second point, we are left with applying effective models. In the hadron resonance gas model at $T = 0$, the nucleons N are supposed to be dominant; other heavier resonances are negligible and the particle density must be equal to the normal nuclear density $n_0 \approx 0.17$ fm^{-3}. For such a degenerate Fermi gas, the chemical potential corresponding to n_0 is $\mu_{ch} \approx 0.979$ GeV.

In HRGM [6, 7, 8, 9], we include all observed resonances up to mass 2 GeV. All particle decays are left away. We use grand canonical ensemble and quantum statistics. Corrections due to van der Waals repulsive interactions and excluded volume have not been taken into account. As we discussed in [6], the effective strong interactions are regarded via including heavy resonances. In the following, we list some thermodynamic expressions for one particle and its anti-particle in the Boltzmann limit.

$$n(T,\mu) \; = \; \frac{g}{\pi^2} T m^2 K_2\left(\frac{m}{T}\right)\, \sinh\left(\frac{\mu}{T}\right), \tag{4}$$

$$s(T,\mu) \; = \; \frac{g}{\pi^2} m^2 \left[m K_3\left(\frac{m}{T}\right)\, \cosh\left(\frac{\mu}{T}\right) - \mu K_2\left(\frac{m}{T}\right)\, \sinh\left(\frac{\mu}{T}\right)\right], \tag{5}$$

$$\frac{\epsilon(T,\mu)}{n(T,\mu)} \; = \; \left(3T + m\frac{K_1\left(\frac{m}{T}\right)}{K_2\left(\frac{m}{T}\right)}\right)\, \coth\left(\frac{\mu}{T}\right), \tag{6}$$

where g is the spin-isopsin degeneracy factor and K_i are the modified Bessel functions. These thermodynamic quantities will be summed over all resonances taken into account. They are related to the chemical equilibration via Eq. (3).

II. THERMODYNAMIC CONDITIONS

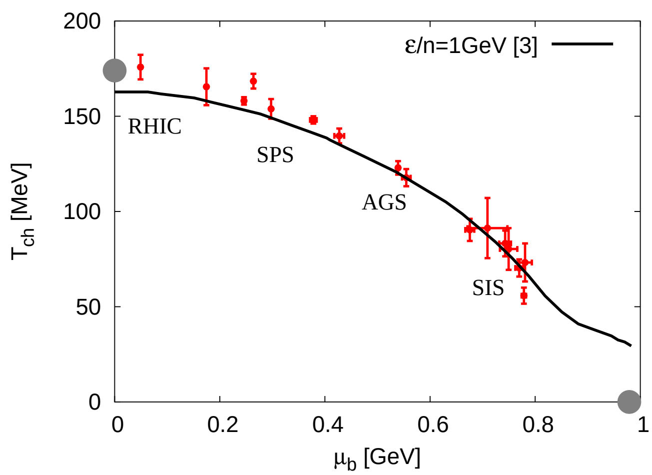

Fig. 1: The freeze-out curve under the condition of constant energy per particle [3]. The points are the freeze-out parameters taken from indicated experiments.

An attempt to describe the freeze-out parameters at different energies is given in [3]. The authors started from phenomenological observations at GSI/SIS energy and found that $\epsilon/n \approx 1\,$GeV. Analytically, the hadron resonance gas at low T can be handled as a non-relativistic gas consisting of degenerate nucleons, so that the ratio in Eq. (6) is $\epsilon/n \approx m_N + 3T/2 \approx 1$ GeV. At relativistic T, the pions and rho-mesons get dominant. The authors applied Eq. (6) in the non-relativistic limit and got almost the same value. In grand canonical ensemble, we get under this condition the curve plotted in Fig. 1.

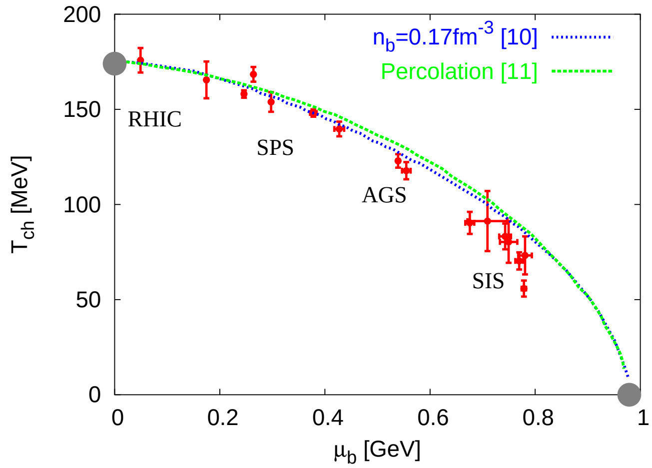

Fig. 2: The freeze-out curve under the condition of constant baryon number density $n_b = 0.17$ fm^{-3} [10]. In calculating the short-dashed curve the percolation theory [11] has been taken into account.

In Ref. [10], a condition of constant baryon number density n_b was imposed. The freeze-out curve is plotted according to $n_b = 0.12$ fm^{-3}. Baryon-baryon and baryon-meson interactions were assumed to drive the freeze-out. This value of n_b, two-third the normal nuclear density, was argued to be dependent on corrections due to van der Waals repulsive interactions. In Fig. 2, we draw our own calculations under the condition $n_b = 0.17$ fm^{-3}, normal nuclear density. We use this value, since we leave away the additional corrections. In comparing our calculations with [10], we see that the assumption of repulsive interactions is not well-founded. Nevertheless, we notice that the value we use is satisfactorily able to reproduce the two endpoints of the freeze-out curve.

Another model we consider is in the framework of percolation theory [11]. At $T = 0$, the freeze-out occurs when the nucleons no longer form interconnected matter. The corresponding density is found ≈ 0.17 fm^{-3} and consequently, $\mu = 0.979\,$GeV. At $\mu = 0$, it has been found that $T_{ch} \approx 175$ MeV for $\gamma_s = 0.5$. γ_s gives the strangeness saturation. $\gamma_s = 1$ is a condition for QGP, but it is unlikely for the hadron matter [6]. The equation which defines the freeze-out at finite T and μ reads

$$n(T,\mu) \;=\; \frac{1.24}{v}\left(1-\frac{n_b(T,\mu)}{n(T,\mu)}\right)+\frac{0.34}{v}\left(\frac{n_b(T,\mu)}{n(T,\mu)}\right), \tag{7}$$

where $v = 4\pi r^3/3$ is the volume of the hadron with radius r. The results are shown in Fig. 2.

We can so far conclude that the last two models [10, 11] are excellent in reproducing the two characteristic endpoints. Both of them apparently overestimate the experimental parameters at BNL/AGS and GSI/SIS energies. Model [3] describes well the experimental results at low energies. It slightly underestimates the BNL/RHIC and CERN/SPS results. Its largest discrepancy is at very low energies. At $\mu = 0.979$ GeV, which corresponds to n_0 at $T = 0$, we find that the system still has a high temperature of about 30 MeV. For $T = 0$, this model results in particle density equals to 25 times n_0.

$$\frac{\epsilon(\mu)}{n(\mu)} = 9gm^4 \left[\frac{\mu}{m}\sqrt{\frac{\mu^2}{m^2}-1}\left(\frac{\mu^2}{m^2}-\frac{1}{2}\right)-\frac{1}{2}\ln\left(\frac{\mu}{m}+\sqrt{\frac{\mu^2}{m^2}-1}\right)\right] \Bigg/ 8\left(\mu^2-m^2\right)^{3/2}. \quad (8)$$

For the degenerate Fermi gas of nucleons at $\mu = 0.979$ GeV, ϵ/n in Eq. (8) becomes 2.89 GeV.

III. ENTROPY DENSITY AT CHEMICAL FREEZE-OUT

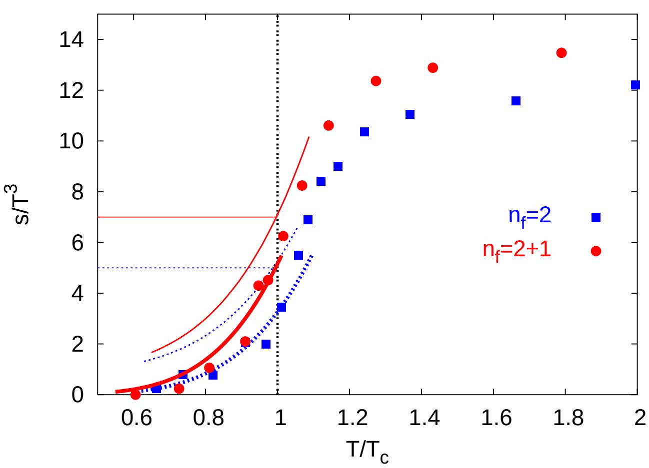

Fig. 3: Lattice QCD results (solid points) of entropy density normalized to T^3 for different quark flavors at $\mu = 0$. The solid curves are HRGM results with re-scaled resonance masses. The *unphysical* heavy resonance masses have been set to be comparable with the quark masses used on lattice. The thin curves give the results corresponding to the physical masses.

As discussed in Sec. I, the condition which guarantees chemical equilibration between reactants and products is the entropy density [4]. At vanishing free energy, the equilibrium entropy gives the amount of energy which can't be used to produce further work. In this

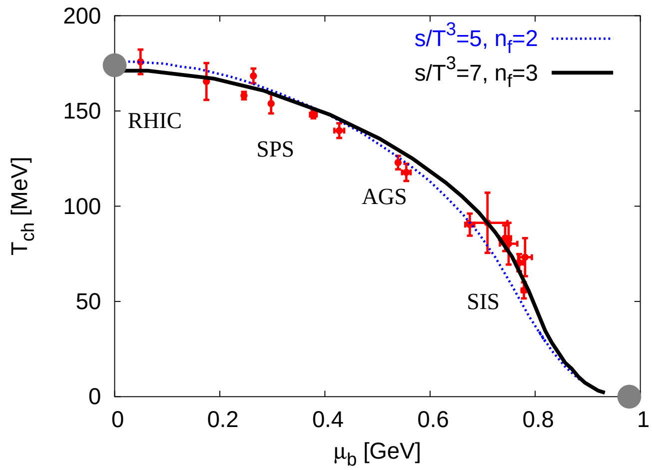

Fig. 4: The freeze-out curve under our condition of constant entropy density. There is almost no difference when changing the flavor number. The two solid circles as well as the experimentally estimated points are very well reproduced.

context, the entropy can be seen as a measure for the degree of sharing and spreading the energy inside the system. The way of distributing the energy is not just an average value. But rather the method that controls the chemical equilibration. i.e. produces no additional work, Eq. (3).

As $T \to 0$ and at $\mu \neq 0$, it is assumed that the hadronic system is in form of degenerate Fermi gas of nucleons. Then from Eq. (5), the entropy density vanishes. At $\mu = 0$ and $T \neq 0$, the system becomes a degenerate Bose gas mainly consists of pions and rho-mesons. In this case, the entropy gets a finite value. The question is: what is the value of equilibrium entropy which can be used at finite T and μ? For the reason that there is no theoretical description of the chemical freezing-out and at small μ the freeze-out and phase transition are *de facto* coincident, we reliant on the lattice value [7, 8, 9, 12].

In Fig. 3, we plot the lattice results of s/T^3 versus T/T_c at $\mu = 0$ [8, 12]. We compare between 2 and $2+1$ quark flavor results. In $2+1$ lattice simulations, two light quarks plus one heavy strange quark are used. Extensive details about simulating lattice results by HRGM are given in [6, 7, 8, 9]. To compare with two quark flavors, we include only the non-strange resonances. For three quark flavors, all resonances are included. The solid curves are HRGM results with re-scaled heavy resonance masses. In order to reasonably compare HRGM with the lattice simulations in which heavy quark masses are used, the resonance masses have to be set at values heavier than the physical ones [7, 8]. The results with the physical masses are given by the thin curves. The two horizontal lines point at s/T^3 at T_c; $s/T^3 = 5$ for $n_f = 2$ and $s/T^3 = 7$ for $n_f = 3$. The normalization with respect to T^3 should not be connected with massless ideal gas. Either the hadrons in HRGM or the quarks on lattice are massive. In lattice QCD simulations, one usually expresses the physical quantities in dimensionless units.

At each value of μ, we calculate T_{ch} according to the given ratio s/T^3 by Eq. (5). The results are plotted in Fig. 4. At $\mu = 0$ and $T \neq 0$, the entropy is finite and as $T \to 0$, the entropy is vanishing. Here, the quantum entropy is entirly disregarded [13, 14, 15, 16, 17, 18]. The intermediate region is very well reproduced by Eq. (5). We see that the two characteristic endpoints as well as all experimentally estimated freeze-out parameters are very well reproduced. Comparing our results with the results shown in Fig. 1 and Fig. 2, one finds that our results are much better in describing the freeze-out parameters.

IV. CONCLUSION

We propose the entropy density as the thermodynamic condition driving the chemical freeze-out. Taking its value from lattice QCD simulations at $\mu = 0$ and assuming it (normalized to T^3) remains constant in the entire μ-axis, we obtain the results shown in Fig. 4. The experimentally estimated freeze-out parameters T_{ch} and μ are very well described under this condition. The two characteristic endpoints of the freeze-out curve are also reproduced. So far we conclude that *the given ratio s/T^3 characterizes very well the final states observed in all heavy-ion experiments*. Increasing the beam energy leads to an increasing in the particle yields. The hadron resonance gas model regards this observation in the way, that the energy increase is represented via including heavier resonances. The energy change with changing the particle number is given by the chemical potential μ. "Second law of thermodynamics" controls this relation. At equilibrium, the amount of energy which produces no work, i.e. at vanishing free energy, is the entropy.

We reviewed the other conditions suggested for the freeze-out curve. As an ideal quantum gas of hadron resonances, we applied the hadron resonance gas model on calculating the freeze-out curve according to these conditions. We compared the results of the models proposed and check their abilities in reproducing the experimentally estimated freeze-out parameters and the two characteristic endpoints (Fig. 1 and Fig. 2). We found that there are different constraints in reproducing the endpoints and fitting the freeze-out parameters.

I thank Rudolf Baier, Tamas Biro, Rajiv Gavai and Helmut Satz for useful discussions.

---

[1] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. **B344**, 43 (1995), nucl-th/9410026.

[2] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. **B365**, 1 (1996), nucl-th/9508020.

[3] J. Cleymans and K. Redlich, Phys. Rev. **C60**, 054908 (1999), nucl-th/9903063.

[4] A. Tawfik (2004), hep-ph/0410329.

[5] F. Becattini, Nucl. Phys. **A702**, 336 (2002), hep-ph/0206203.

[6] A. Tawfik (2004), in progress.

[7] F. Karsch, K. Redlich, and A. Tawfik, Eur. Phys. J. **C29**, 549 (2003), hep-ph/0303108.

[8] F. Karsch, K. Redlich, and A. Tawfik, Phys. Lett. **B571**, 67 (2003), hep-ph/0306208.

[9] K. Redlich, F. Karsch, and A. Tawfik, J. Phys. **G30**, S1271 (2004), nucl-th/0404009.

[10] P. Braun-Munzinger and J. Stachel, J. Phys. **G28**, 1971 (2002), nucl-th/0112051.

[11] V. Magas and H. Satz, Eur. Phys. J. **C32**, 115 (2003), hep-ph/0308155.

[12] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. **B605**, 579 (2001), hep-lat/0012023.

[13] D. E. Miller and A. Tawfik (2003), hep-ph/0308192.

[14] D. E. Miller and A. Tawfik (2003), hep-ph/0309139.

[15] D. E. Miller and A. Tawfik (2003), hep-ph/0312368.

[16] D. E. Miller and A. Tawfik, J. Phys. **G30**, 731 (2004), hep-ph/0402296.

[17] S. Hamieh and A. Tawfik (2004), hep-ph/0404246.

[18] D. E. Miller and A. Tawfik, Acta Phys. Polon. **B35**, 2165 (2004), hep-ph/0405175.