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Isospin quantum number of $D_{s0}^+(2317)$

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The $D_s^+\pi^0$ and $D_s^{*+}\gamma$ decays of the $D_{s0}^+(2317)$ are studied under the vector-meson dominance hypothesis. Its assignment to the four-quark meson with $(I, I_3) = (1, 0)$ is consistent with experimental data on these decays, while those to $I = 0$ states (a four-quark and a conventional $\{c\bar{s}\}$) are not favored.

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Recently, a narrow $D_s^+\pi^0$ resonance with the mass $\simeq$ 2317 MeV has been observed [1, 2], and its assignments to various states have been proposed [3, 4, 5, 6] in addition to the ordinary scalar $\{c\bar{s}\}$ [7] which is the chiral partner of D_s^+ [8], prior to the observation. However, its isospin quantum number seems to be still not definite. In this short note, therefore, we will study the $D_{s0}^+(2317) \to D_s^{*+}\gamma$ and $D_s^+\pi^0$ decays, and demonstrate that the severest experimental constraint on the ratio of the rates [2],

$$\frac{\Gamma(D_{s0}^+(2317) \to D_s^{*+}\gamma)}{\Gamma(D_{s0}^+(2317) \to D_s^+\pi^0)} < 0.052, \qquad (1)$$

can be satisfied easily, if the $D_{s0}^+(2317)$ is assigned to the scalar four-quark meson with $(I, I_3) = (1, 0)$, $\hat{F}_I^+ \sim [cn][\bar{s}\bar{n}]_{I=1}$ $(n = u, d)$. It is hard to reconcile the assignment to an $I = 0$ state (the conventional scalar $\{c\bar{s}\}$ or the scalar four-quark, $\hat{F}_0^+ \sim [cn][\bar{s}\bar{n}]_{I=0}$) with Eq. (1), where the square brackets imply that the wave-functions are anti-symmetric under exchange of the flavors between them (see Ref. [4] about detailed notation of four-quark mesons). The possible existence of such four-quark states has been also suggested for the light-flavor mesons (the nonet of $[qq][\bar{q}\bar{q}]$: $\hat{\delta}^s$, $\hat{\sigma}^s$, $\hat{\sigma}$, $\hat{\kappa}$) [9], which would correspond to the observed scalar mesons [10], $a_0(980)$, $f_0(980)$, $\sigma(600)$ and $\kappa(800)$ [11], respectively.

If the $D_{s0}^+(2317)$ is assigned to the $\hat{F}_I^+$, its decay into the $D_s^+\pi^0$ can proceed through the isospin-conserving strong interactions. As a result, the rate is expected to be much larger than that of the radiative $\hat{F}_I^+ \to D_s^{*+}\gamma$ decay, and thus the constraint (1) can be quite naturally satisfied. The origin of its narrow width can be understood by small overlapping of wave-functions with respect to the color and spin [5]. It will be seen more explicitly below. The amplitude for the main decay, $\hat{F}_I^+ \to D_s^+\pi^0$, can be approximated in the form,

$$M(\hat{F}_I^+ \to D_s^+\pi^0) \simeq \left(\frac{m_{\hat{F}_I}^2 - m_{D_s}^2}{f_{\pi^0}}\right)\langle D_s^+|A_{\pi^0}|\hat{F}_I^+\rangle, \quad (2)$$

by using a hard pion technique with the PCAC in the infinite momentum frame (IMF) [12, 13], where A_π is the axial counterpart of the isospin I and f_{π^0} the decay constant of π^0. Then, the decay rate is defined as

$$\Gamma(\hat{F}_I^+ \to D_s^+\pi^0) = \frac{q_c \sum_{spin} |M(\hat{F}_I^+ \to D_s^+\pi^0)|^2}{8\pi(2J_A + 1)m_{\hat{F}_I}^2} \qquad (3)$$

with J_A and q_c being the spin of $\hat{F}_I^+$ and the center-of-mass momentum of the final mesons, respectively, and the summation is taken for all spin states. The *asymptotic* matrix element of A_π (matrix element of A_π taken between single hadron states with infinite momentum), $\langle B|A_\pi|A\rangle$, gives the dimensionless $AB\pi$ coupling strength. Here we compare the $\hat{F}_I^+ \to D_s^+\pi^0$ with the $\hat{\delta}^{s+} \to \eta\pi^+$ by using the *asymptotic* $SU_f(4)$ symmetry (roughly speaking, $SU_f(4)$ symmetry of the asymptotic matrix elements) [13]. The amplitude for the $\hat{\delta}^{s+} \to \eta\pi^+$ is obtained by replacing ($\hat{F}_I^+$, D_s^+, π^0) by ($\hat{\delta}^{s+}$, η, π^+) in Eq. (2). Taking the usual η-η' mixing [10], we obtain $\eta = \cos\Theta \cdot \eta^n - \sin\Theta \cdot \eta^s$, where η^n and η^s are the $\{n\bar{n}\}$ and $\{s\bar{s}\}$ components of η, respectively, and $\Theta = \chi + \theta_P$ with $\cos\chi = 1/\sqrt{3}$ and the η-η' mixing angle $\theta_P \simeq -20°$. Since only the η^s component survives in the matrix element, $\langle\eta|A_{\pi^-}|\hat{\delta}^{s+}\rangle$, due to the OZI rule, we have $\langle\eta|A_{\pi^-}|\hat{\delta}^{s+}\rangle = -\sin\Theta \cdot \langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle$. A *naive* application of the asymptotic $SU_f(4)$ symmetry leads to $\sqrt{2}\langle D_s^+|A_{\pi^0}|\hat{F}_I^+\rangle = \langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle$. This estimation would be, however, too naive by the following two reasons: (i) The asymptotic $SU_f(4)$ symmetry would overestimate by about 20 % the size of asymptotic matrix elements involving a charmed meson state [6]. (ii) In the color and spin spaces, the crossing matrices of $\hat{F}_I^+ \sim |[cn]_{\mathbf{\bar{3}_c}}^{\mathbf{1}}[\bar{s}\bar{n}]_{\mathbf{3_c}}^{\mathbf{1}}\rangle$ with the specific color-singlet state composed of color- and spin-less $\{q\bar{q}\}$ pairs, $|\{q\bar{q}\}_{\mathbf{1_c}}^{\mathbf{1}}\{q\bar{q}\}_{\mathbf{1_c}}^{\mathbf{1}}\rangle$, is much smaller than one [9]. Here the sub- and super-scripts denote color and spin multiplets, respectively. However, this is not true for the light four-quark mesons as the $\hat{\delta}^{s+}$ (at lower energy scale), because soft-gluon exchanges in the system reshuffle such a specific color-singlet state and cause a configuration mixing. As a result, the crossing matrices with the $\hat{\delta}^{s+}$ approach one. Therefore, we modify the prior asymptotic $SU_f(4)$ relations by introducing a

parameter

$$\beta = \frac{\sqrt{2}\langle D_s^+|A_{\pi^0}|\hat{F}_I^+\rangle}{\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle}. \quad (4)$$

In the naive limit of the asymptotic $SU_f(4)$ symmetry, $\beta = 1$ as seen above. We hereafter call the case of $\beta \neq 1$ the modified asymptotic $SU_f(4)$ symmetry. For simplicity, we consider the limiting case that there is no configuration mixing for the $\hat{F}_I^+$, while there is no suppression of the crossing matrices for the $\hat{\delta}^{s+}$ (a maximum mixing of the color and spin configurations), i.e., $|\beta|^2 = \frac{1}{12}$ [9]. Inserting $|\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle| \sim 0.8$ obtained from the tentative value, $\Gamma(a_0 \to \eta\pi^+)_{\rm exp} \simeq 70$ MeV (the measured rate is $50 \sim 100$ MeV [10]), we obtain

$$\Gamma(\hat{F}_I^+ \to D_s^+\pi^0) \sim 9\,{\rm MeV}. \quad (5)$$

Here the observed $a_0(980)$ has been assigned to the $\hat{\delta}^s$ and $|\beta|^2 = \frac{1}{12}$ has been taken. This result is a little larger than the measured width of the $D_{s0}^+(2317)$ [1, 2] but still compatible with the data if the $SU_f(4)$ symmetry breaking is taken into account as mentioned in (i) above.

When the $D_{s0}^+(2317)$ is assigned to an $I = 0$ state (the scalar $\{c\bar{s}\}$ or the $\hat{F}_0^+$), however, it is difficult to reconcile with Eq. (1), since isospin violating phenomena, e.g., the difference of d-u quark masses, $\Delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$, $\Delta m_K = m_{K^0} - m_{K^+}$, etc., have been considered as the second order effects of QED [14] and the π^0-η mixing [15] could be also of the same origin. Therefore, the isospin non-conserving decay of $D_{s0}^+(2317) \to D_s^+\pi^0$, which is assumed to proceed through the π^0-η mixing, is expected to be (much) weaker than the radiative decay in contrast with the arguments based on specific models [16].

Before discussing isospin non-conserving decays of the D_s^{*+} and $D_{s0}^+(2317)$ into $D_s^+\pi^0$, we first investigate their radiative decays under the vector-meson dominance (VMD) hypothesis [17] with the flavor $SU_f(4)$ symmetry for the strong vertices and next compare the results with those of the $D_s^+\pi^0$ decays. Since the VMD with the ideal ω-ϕ mixing and the flavor $SU_f(3)$ for the strong vertices works fairly well in the radiative decays of light vector-mesons [18], we will extend it to the system containing charm quarks below. The $V \to P\gamma$ amplitude can be written in the form,

$$M(V \to P\gamma) = \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu}(V) F_{\alpha\beta}(\gamma) A(V \to P\gamma), \quad (6)$$

where V, P and $A(V \to P\gamma)$ denote a vector meson, a pseudoscalar meson and the $VP\gamma$ coupling strength, respectively, and $G_{\mu\nu}(V)$ and $F_{\alpha\beta}(\gamma)$ are the field strengths of a vector meson (V) and a photon (γ), respectively. Under the VMD, the $A(V \to P\gamma)$ can be approximated by

$$A(V \to P\gamma) \simeq \sum_{V'=\rho^0,\,\omega,\,\phi,\,\psi} \left[\frac{X_{V'}(0)}{m_{V'}^2}\right] A(V \to PV'), \quad (7)$$

where $X_V(0)$ is the γV coupling strength on the photon mass-shell. (The photon-momentum-square dependence of X_V has been studied in Ref. [18].) The values of $X_V(0)$'s have been estimated from the analyses in photoproductions of vector mesons on various nuclei [19]. For the ψ photoproduction, both the measured differential cross section at $t = 0$ and the ψN total cross section estimated from the A-dependence of photoproduction cross sections still have large uncertainties, where N denotes a nucleon. The results are $X_\rho(0) = 0.033 \pm 0.003$ GeV^2, $X_\omega(0) = 0.011 \pm 0.001$ GeV^2, $X_\phi(0) = -0.018 \pm 0.004$ GeV^2 and $X_\psi(0) \sim 0.054$ GeV^2, where the last one has been obtained from $d\sigma(\gamma N \to \psi N)/dt|_{t=0} \simeq 20$ nb/GeV^2 and $\sigma_T(\psi N) = 3.5 \pm 0.8$ mb [20] for the ψNtotal cross section. The values of $X_\phi(0)$ and $X_\psi(0)$ are considerably lower than those of $X_\phi(m_\phi^2)$ and $X_\psi(m_\psi^2)$ estimated from the measured rates for the lepton-pair decays of ϕ and ψ, respectively. The VPV' coupling strength can be estimated from the measured rate for the $\omega \to \pi^0\gamma$ decay: putting $(V, P, V') = (\omega, \pi^0, \rho^0)$ in Eq. (7) and inserting the above value of $X_\rho(0)$, we have

$$|A(\omega \to \pi^0\rho^0)| \simeq 18\,{\rm GeV}^{-1}, \quad (8)$$

where we have used $\Gamma(\omega \to \pi^0\gamma)_{\rm exp} = 0.734 \pm 0.035$ MeV [10].

To estimate uncertainties arising from the VMD and the $SU_f(4)$ symmetry for VPV' vertices, we focus on the $D^* \to D\gamma$ decays. The amplitudes, $A(D^* \to D\gamma)$'s, are given by the ρ^0, ω and ψ-meson poles. Using the $SU_f(4)$ relations, $2A(D^{*0} \to D^0\rho^0) = 2A(D^{*0} \to D^0\omega) = \sqrt{2}A(D^{*0} \to D^0\psi) = -2A(D^{*+} \to D^+\rho^0) = 2A(D^{*+} \to D^+\omega) = \sqrt{2}A(D^{*+} \to D^+\psi) = \sqrt{2}A(D^{*0} \to D^0\psi) = A(\omega \to \pi^0\rho^0)$, and Eq. (8), we obtain $\Gamma(D^{*+} \to D^+\gamma) \sim 2.4$ keV and $\Gamma(D^{*0} \to D^0\gamma) \sim 19$ keV. The former is compatible with the measured $\Gamma(D^{*+} \to D^+\gamma)_{\rm exp} \simeq 1.5$ keV (with $\sim 30\,\%$ errors) by taking account of the $SU_f(4)$ symmetry breaking and the latter satisfies $\Gamma(D^{*0} \to D^0\gamma)_{\rm exp} < 800$ keV [10].

Here for later discussion, we consider the $D_s^{*+} \to D_s^+\gamma$ decay to compare the result with the $D_s^{*+} \to D_s^+\pi^0$. The amplitude is given by ϕ and ψ-meson poles. Using the $SU_f(4)$ symmetry relations, $A(D_s^{*+} \to D_s^+\phi) = A(D_s^{*+} \to D_s^+\psi) = \sqrt{\frac{1}{2}}A(\omega \to \pi^0\rho^0)$, and Eq. (8), we get

$$\Gamma(D_s^{*+} \to D_s^+\gamma) \sim 0.8\,{\rm keV}, \quad (9)$$

which satisfies $\Gamma(D_s^{*+} \to D_s^+\gamma)_{\rm exp} < 1.8$ MeV [10].

Similarly, the amplitude for the radiative decay of scalar mesons, $S \to V\gamma$, can be written in the form,

$$M(S \to V\gamma) = G_{\mu\nu}(V) F^{\mu\nu}(\gamma) A(S \to V\gamma). \quad (10)$$

Under the VMD, the above $A(S \to V\gamma)$ is given by

$$A(S \to V\gamma) \simeq \sum_{V'=\rho^0,\,\omega,\,\phi,\,\psi} \left[\frac{X_{V'}(0)}{m_{V'}^2}\right] A(S \to VV'). \quad (11)$$

When the $D_{s0}^+(2317)$ is assigned to the $\hat{F}_I^+ \sim [cn][\bar{s}\bar{n}]_{I=1}$, the amplitude $A(\hat{F}_I^+ \to D_s^{*+}\gamma)$ is dominated by the ρ^0-meson pole because of the OZI rule,

$$A(\hat{F}_I^+ \to D_s^{*+}\gamma) \simeq \left[\frac{X_\rho(0)}{m_\rho^2}\right] A(\hat{F}_I^+ \to D_s^{*+}\rho^0). \quad (12)$$

Using the modified $SU_f(4)$ symmetry for the SVV' vertices, we obtain $2A(\hat{F}_I^+ \to D_s^{*+}\rho^0) = A(\phi \to \hat{\delta}^{s0}\rho^0)\beta'$, where β' denotes the suppression factor (like the β before) arising from the overlapping of the color and spin wave-functions. We again assume the limiting case, i.e., the full suppression for $A(\hat{F}_I^+ \to D_s^{*+}\rho^0)$ and no suppression for $A(\phi \to \hat{\delta}^{s0}\rho^0)$, so-called the fall-apart amplitude. However, since in this case we must take the $|\{q\bar{q}\}\{q\bar{q}\}\rangle$ as a pair of vector states, $|\beta'|^2 = \frac{1}{4}$ [9]. Inserting

$$|A(\phi \to \hat{\delta}^{s0}\rho^0)| \simeq 0.020\,\mathrm{MeV}^{-1} \quad (13)$$

obtained from $\Gamma(\phi \to a_0(980)\gamma)_{\rm exp} = 0.32 \pm 0.03$ keV [10] and $|\beta'|^2 = \frac{1}{4}$, we have

$$\Gamma(\hat{F}_I^+ \to D_s^{*+}\gamma) \simeq 45\,\mathrm{keV}. \quad (14)$$

Therefore, using Eq.(5), the ratio reads

$$\frac{\Gamma(\hat{F}_I^+ \to D_s^{*+}\gamma)}{\Gamma(\hat{F}_I^+ \to D_s^+\pi^0)} \sim 0.005, \quad (15)$$

which satisfies well the constraint (1).

When the $D_{s0}^+(2317)$ is assigned to the conventional scalar F_0^+, the amplitude $A(F_0^+ \to D_s^{*+}\gamma)$ is given by

$$A(F_0^+ \to D_s^{*+}\gamma) \simeq \left[\frac{X_\phi(0)}{m_\phi^2}\right] A(F_0^+ \to D_s^{*+}\phi) + \left[\frac{X_\psi(0)}{m_\psi^2}\right] A(F_0^+ \to D_s^{*+}\psi) \quad (16)$$

under the VMD. The $SU_f(4)$ symmetry for the SVV' vertices leads to $2A(F_0^+ \to D_s^{*+}\phi) = 2A(F_0^+ \to D_s^{*+}\psi) = A(\chi_{c0} \to \psi\psi)$. From the measured $\Gamma(\chi_{c0} \to \psi\gamma)_{\rm exp} = 182 \pm 15$ keV [10], we have $|A(\chi_{c0} \to \psi\psi)| \simeq 0.102$ MeV^{-1}, and thus

$$\Gamma(F_0^+ \to D_s^{*+}\gamma) \simeq 50 \text{ keV}. \quad (17)$$

When the $D_{s0}^+(2317)$ is assigned to the iso-singlet scalar four-quark meson, $\hat{F}_0^+ \sim [cn][\bar{s}\bar{n}]_{I=0}$, the amplitude $A(\hat{F}_0^+ \to D_s^{*+}\gamma)$ is given by the ω-meson pole,

$$A(\hat{F}_0^+ \to D_s^{*+}\gamma) \simeq \left[\frac{X_\omega(0)}{m_\omega^2}\right] A(\hat{F}_0^+ \to D_s^{*+}\omega) \quad (18)$$

under the VMD. Using the modified $SU_f(4)$ symmetry with the suppression factor β', $A(\hat{F}_0^+ \to D_s^{*+}\omega) = A(\phi \to \hat{\delta}^{s0}\rho^0)\beta'$. Inserting this relation with Eq. (13) and $|\beta'|^2 = \frac{1}{4}$ into Eq. (18), we obtain

$$\Gamma(\hat{F}_0^+ \to D_s^{*+}\gamma) \simeq 4.7 \text{ keV}, \quad (19)$$

which is smaller by one order of magnitude than Eq. (14) since $X_\omega(0) \simeq \frac{1}{3}X_\rho(0)$.

Next we discuss the isospin non-conserving decays. The amplitude for the $D_s^{*+} \to D_s^+\pi^0$ is approximately written in the form,

$$M(D_s^{*+} \to D_s^+\pi^0) \simeq \left(\frac{m_{D_s^*}^2 - m_{D_s}^2}{f_{\pi^0}}\right) \langle D_s^+|A_{\pi^0}|D_s^{*+}\rangle \quad (20)$$

as in Eq. (2). Here we assume that the isospin non-conservation is caused by the η-π^0 mixing, i.e., $A_{\pi^0} \to A_{\pi^0} + \epsilon A_\eta$, ($|\epsilon| \ll 1$), so that we replace $\langle D_s^+|A_{\pi^0}|D_s^{*+}\rangle$ by $\epsilon\langle D_s^+|A_\eta|D_s^{*+}\rangle$. The axial charge A_η is given by $A_\eta = \cos\Theta \cdot A_{\eta^n} - \sin\Theta \cdot A_{\eta^s}$, where A_{η^n} and A_{η^s} are the $\{n\bar{n}\}$ and $\{s\bar{s}\}$ components of A_η, respectively. In the asymptotic matrix element of A_η, only the matrix element of A_{η^s} can survive due to the OZI rule and the asymptotic $SU_f(4)$ symmetry leads to $\langle D_s^+|A_{\eta^s}|D_s^{*+}\rangle = \frac{1}{2}\langle\pi^+|A_{\pi^+}|\rho^0\rangle$. From these relations,

$$\langle D_s^+|A_{\pi^0}|D_s^{*+}\rangle = -\frac{1}{2}\epsilon\sin\Theta \cdot \langle\pi^+|A_{\pi^+}|\rho^0\rangle. \quad (21)$$

The size of the $\langle\pi^+|A_{\pi^+}|\rho^0\rangle$ can be estimated to be $|\langle\pi^+|A_{\pi^+}|\rho^0\rangle| \simeq 1.0$ [12] from the measured rate, $\Gamma(\rho \to \pi\pi)_{\rm exp} \simeq 150$ MeV [10]. Inserting Eq. (21) with $\Theta \simeq 35^\circ$ ($\theta_P \simeq -20^\circ$) as before into Eq. (20) and using the measured branching ratios for the $D_s^{*+} \to D_s^+\pi^0, D_s^+\gamma$ decays [10] and Eq. (9), we get

$$|\epsilon| \sim 1.0 \times 10^{-2}, \quad (22)$$

which is compatible with the second order QED effect as expected before.

With the help of the value of ϵ, we finally consider two cases of the isospin non-conserving decay, $D_{s0}^+(2317) \to D_s^+\pi^0$; (i) $D_{s0}^+(2317)$ as the conventional scalar $F_0^+ \sim \{c\bar{s}\}$ and (ii) $D_{s0}^+(2317)$ as the scalar four-quark $\hat{F}_0^+ \sim [cn][\bar{s}\bar{n}]_{I=0}$. The amplitudes for the S^+ ($= F_0^+$ or $\hat{F}_0^+$) $\to D_s^+\pi^0$ decays can be approximated in the form,

$$M(S^+ \to D_s^+\pi^0) \simeq \left(\frac{m_S^2 - m_{D_s}^2}{f_{\pi^0}}\right) \langle D_s^+|A_{\pi^0}|S^+\rangle \quad (23)$$

as in Eq. (2). Since such a decay is assumed to proceed through the π^0-η mixing as discussed before, we replace the matrix elements, $\langle D_s^+|A_{\pi^0}|F_0^+\rangle$ and $\langle D_s^+|A_{\pi^0}|\hat{F}_0^+\rangle$, by the OZI-allowed $-\epsilon\sin\Theta \cdot \langle D_s^+|A_{\eta^s}|F_0^+\rangle$ and $\epsilon\cos\Theta \cdot \langle D_s^+|A_{\eta^n}|\hat{F}_0^+\rangle$, respectively.

(i) The asymptotic $SU_f(4)$ symmetry leads to $\langle D_s^+|A_{\eta^s}|F_0^+\rangle = \langle K^+|A_{\pi^+}|K_0^{*0}(1430)\rangle$, whose size is estimated to be $|\langle K^+|A_{\pi^+}|K_0^{*0}(1430)\rangle| \simeq 0.29$ from the experimental data, $\Gamma(K_0^{*0} \to K^+\pi^-)_{\rm exp} = 182 \pm 24$

MeV [10]. Here we have assumed that $K_0^{*0}(1430)$ is the conventional $^3P_0\,\{d\bar{s}\}$ state [10]. Using the above result on the asymptotic matrix elements, the value of $|\epsilon|$ in Eq. (22) and $\theta_P = -20^\circ$, we have

$$\Gamma(F_0^+ \to D_s^+\pi^0) \sim 0.6 \text{ keV}, \tag{24}$$

which is much smaller than Eq. (17) as expected.
(ii) The modified asymptotic $SU_f(4)$ symmetry leads to $2\langle D_s^+|A_{\eta^n}|\hat{F}_0^+\rangle = \langle\pi^+|A_{\eta^s}|\hat{\delta}^{s+}\rangle\beta$. Taking $|\beta|^2 = \frac{1}{12}$ and $|\langle\pi^+|A_{\eta^s}|\hat{\delta}^{s+}\rangle| = \sqrt{1/2}|\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle| \sim 0.6$ estimated before, we obtain

$$\Gamma(\hat{F}_0^+ \to D_s^+\pi^0) \sim 0.7 \text{ keV}, \tag{25}$$

which is lower than Eq. (19). Eventually, the ratios of (17) to (24), and (19) to (25) exceed one significantly and thus we conclude that the assignments of the $D_{s0}^+(2317)$ to the $I = 0$ states are not favored.

In summary, we have studied the $D_{s0}^+(2317) \to D_s^+\pi^0$, $D_s^{*+}\gamma$ decays, and have seen the followings. (a) The assignment of the $D_{s0}^+(2317)$ to the iso-triplet four-quark meson, $\hat{F}_I^+$, is consistent with the experimental constraint Eq. (1) as seen in Eq. (15). (b) The assignments of the $D_{s0}^+(2317)$ to the iso-singlet mesons (the four-quark $\hat{F}_0^+$ and the conventional F_0^+) are inconsistent with the measured constraint. This is quite natural because the isospin non-conservation has been expected to be of the second order effect of QED. Thus, the assignment of the $D_{s0}^+(2317)$ to the four-quark meson, $[qq][\bar{q}\bar{q}]$, with $(I, I_3) = (1, 0)$ is favored by the measurements of its $D_s^+\pi^0$ and $D_s^{*+}\gamma$ decays, while its neutral and doubly charged partners have not been observed [21]. The observation, however, does not seem to reject promptly the assignment of the $\hat{F}_I$ with $I = 1$ to the $D_{s0}^+(2317)$. It is because it will be closely associated with the production mechanism of the $\hat{F}_I$ mesons, whether the different charged states should be equally observed or not in the experimental processes. The problem with the production mechanism of the $D_{s0}^{0,+,++}$ will be left as one of our future subjects.

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[1] B. Aubert et al., the BABAR Collaboration, Phys. Rev. Lett. **90**, 242001 (2003); K. Abe et al., the BELLE Collaboration, *ibid.* **91**, 262002 (2003).

[2] D. Besson et al., the CLEO Collaboration, Phys. Rev. D **68**, 032002 (2003).

[3] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003); H.-Y. Chen and W.-S. Hou, Phys. Lett. B **566**, 193 (2003); T. E. Browder, S. Pakvasa and A. A. Petrov, *ibid.* **578**, 365 (2004); E. E. Kolomeitzev and M. F. M. Lutz, *ibid.* **582**, 39 (2004); E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003); J. Hofmann and M. F. M. Lutz, Nucl. Phys. A **733**, 142 (2004).

[4] K. Terasaki, Phys. Rev. D **68**, 011501(R) (2003).

[5] K. Terasaki, hep-ph/0309119; hep-ph/0309279.

[6] K. Terasaki, hep-ph/0405146.

[7] A. De Rújura, H. Georgi and S. L. Glashow, Phys. Rev. Lett. **37**, 785 (1976).

[8] M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D **48**, 4370 (1993); W. A. Bardeen and C. T. Hill, *ibid.* **49**, 409 (1994).

[9] R. L. Jaffe, Phys. Rev. D **15**, 267 and 281 (1977).

[10] S. Eidelman et al., the Particle Data Group, Phys. Lett. B **592**, 1 (2004).

[11] E. M. Aitala et al., the E791 collaboration, Phys. Rev. Lett. **89**, 121801 (2002).

[12] K. Terasaki, S. Oneda, and T. Tanuma, Phys. Rev. D **29**, 456 (1984).

[13] S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. No. 82, 1 (1985) and references therein.

[14] F. E. Close, *An Introduction to Quarks and Partons* (Academic, New York, 1979).

[15] P. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).

[16] S. Godfrey, Phys. Lett. B **568**, 254 (2003); P. Colangelo and F. De Fazio, *ibid.* **570**, 180 (2003); W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D **68**, 054024 (2003); Ya. I. Azimov and K. Goeke, hep-ph/0403082.

[17] See, for example, J. J. Sakurai, *Currents and Mesons* (The University of Chicago Press, Chicago, 1969) and references therein.

[18] K. Terasaki, Lett. Nuovo Cimento **31**, 457 (1981); IL Nuovo Cimento **66A**, 475 (1981). The data used in these articles now should be updated.

[19] D. W. G. S. Leith, *Electromagnetic Interactions of Hadrons*, edited by A. Donnachie and G. Shaw (Plenum Press, New York, 1978), p. 345.

[20] S. D. Holmes, W. Lee and J. E. Wiss, Ann. Rev. Nucl. Part. Sci. **35**, 397 (1985).

[21] B. Aubert et al., the BABAR Collaboration, hep-ex/0408067.