

NUMERICAL MODELLING OF QUANTUM STATISTICS IN HIGH-ENERGY PHYSICS

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Abstract

Numerical modelling of quantum effects caused by bosonic or fermionic character of secondaries produced in high energy collisions of different sorts is at the moment still far from being established. In what follows we propose novel numerical method of *modelling* Bose-Einstein correlations (BEC) observed among identical (bosonic) particles produced in such reactions. We argue that the most natural approach is to work directly in the momentum space of produced secondaries in which the Bose statistics reveals itself in their tendency to bunch in a specific way in the available phase space. Fermionic particles can also be treated in similar fashion.

The multiparticle production processes consist substantial part of the high energy collisions and are of considerable theoretical interest. Unfortunately their description is so far available only by means of numerical Monte Carlo codes based to some extent on modern theoretical ideas but otherwise remaining purely phenomenological [1]. They are build in such manner as to describe as close as possible the complicated final state of such reaction, cf., Fig. 1. However, using classical (positive defined) probabilities as their basic tool such MC codes cannot directly describe some observed features, which are connected with Bose-Einstein (BE) (or Fermi-Dirac (FD)) statistics of produced secondaries (in case they are identical and occupy almost the same parts of the phase space defined by the uncertainty relation, see Fig. 1). When two (or more) identical particles of the same kind are observed, their common wave function should be symmetrized (for BE statistics) or antisymmetrized (for FD statistics) what results in characteristic shapes of (two particle, for example) correlation function $C_2(Q = |p_1 - p_2|) = N(p_1, p_2) / [N(p_1) \cdot N(p_2)]$, cf., Fig. 2.

Referring for details of Bose-Einstein correlations (BEC) to the literature (cf., for example, [2] and references therein) let us concentrate here directly on the problem of their *proper numerical modelling*, i.e., such in which the bosonic character of secondaries produced in hadronization process are going to be accounted

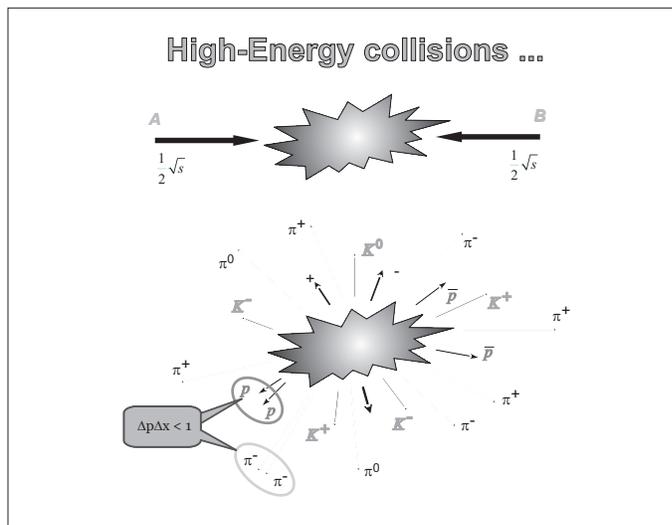


Figure 1: Schematic view of high energy collision resulting in production of many particles of different statistics.

for from the very beginning. This problem was so far considered only in [3] (using statistical approach based on information theory approach, cf., however, also [4]). All other approaches, which claim to model BEC numerically [5], simply add to the outcomes of existing MC codes [1] some *afterburners*, which modify them in a suitable way to be able fit the BEC data. Such approach inevitably leads to such unwanted features as violation of energy-momentum conservation or changes in the original (i.e., obtained directly from MC code) multiparticle spectra.

In [6] we have proposed afterburner free from such unwanted effects. It was based on different concept of introducing quantum mechanical (QM) effects in the otherwise purely probabilistic distributions than those proposed in [7]. Namely, each MC code provides us usually with a given number of particles, each one endowed with either (+) or (-) or (0) charge and with well defined spatio-temporal position and energy-momentum. But experiment provides us information on only the first and last characteristics. The spatio-temporal information is not available directly. In fact, the universal hope expressed in [2, 5] is that precisely this information can be deduced from the previous two via the measured BEC. Our reasoning was as follows: (i) BEC phenomenon is of QM origin therefore one has to introduce in the otherwise purely classical distributions provided by MCG the new element mimicking QM uncertainties; (ii) it cannot be done with energy-momenta because they are measured and therefore fixed; (iii) the next candidate, i.e., spatio-temporal characteristics can be changed but it was already done in [7, 5]; (iv) one is thus left with charges and in [6] we have simply assigned (on event-by-event basis) new charges to the particles from MCG conserving, however, the original multiplicities of (+/-/0). This has been done in such way as to make particles

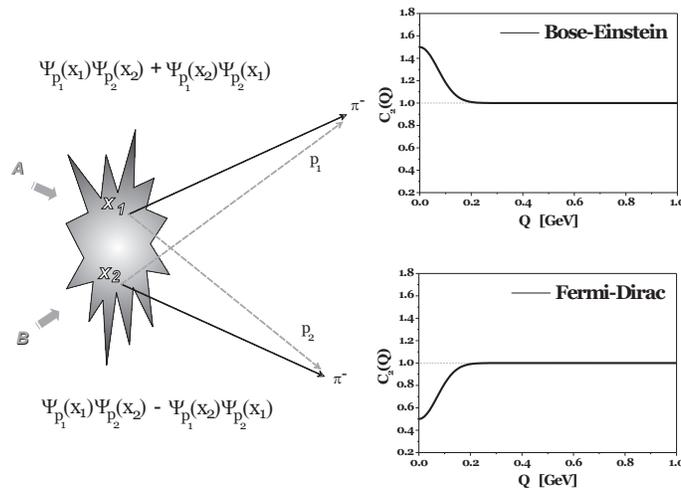


Figure 2: Example of BE and FD statistics (left panel) and two particle correlation functions they lead to (right panels).

of the same charge to be located maximally near to each other in the phase space exploring for this natural fluctuations in spatio-temporal and energy-momentum characteristic of the outcome of MCG. The advantages of such approach are: (a) energy-momentum is automatically conserved and multiparticle distributions are not modified and (b) it is applicable already on the level of each event provided by MCG (not only, as some of propositions of [5] only to all events). However, the new assignment of charges introduces a profound change in the structure of the original MCG. Generally speaking (cf. [6] for details) it requires introduction of bunchings of particles of the same charge.

This observation will be the cornerstone of our new proposition. Let us first remind that idea of bunching of particles as quantum statistical (QS) effect is not the new one [8]. It was used in connection with BEC for the first time in [9] and then was a basis of the so called *clan model* of multiparticle distributions leading in natural way to their negative binomial (NB) form observed in experiment [10]. It was then again introduced in the realm of BEC in [11] and [3, 4]. Because our motivation comes basically from [3] let us outline shortly its basic points. It deals with the problem of how to distribute in a least biased way a given number of bosonic secondaries, $\langle n \rangle = \langle n^{(+)} \rangle + \langle n^{(-)} \rangle + \langle n^{(0)} \rangle$, $\langle n^{(+)} \rangle = \langle n^{(-)} \rangle = \langle n^{(0)} \rangle$. Using information theory approach (cf., [12]) their rapidity distribution was obtained in form of grand partition function with temperature T and chemical potential μ . In addition, the rapidity space was divided into *cells* of equal size δy each (it was fitted parameter). It turned out that whereas the very fact of existence of such cells was enough to obtain reasonably good multiparticle distributions, $P(n)$, (actually, in the NB-like form), their size, δy , was crucial for obtaining the characteristic form of the 2-body BEC function $C_2(Q = |p_i - p_j|)$ (peaked and greater than

unity at $Q = 0$ and then decreasing in a characteristic way towards $C_2 = 1$ for large values of Q , see Fig. 2) out of which one usually deduces the spatio-temporal characteristics of the hadronization source [2] (see [3] for more details). The outcome was obvious: to get C_2 peaked and greater than unity at $Q = 0$ and then decreasing in a characteristic way towards $C_2 = 1$ for large values of Q one must have particles located in cells in phase space which are of nonzero size. It means then that from C_2 one gets not the size of the hadronizing source but only size of the emitting cell, in [3] $R \sim 1/\delta y$, cf. [13]. In the quantum field theoretical formulation of BEC this directly corresponds to the necessity of replacing delta functions in commutator relations by a well defined peaked functions introducing in this way same dimensional scale to be obtained from fits to data [14]. This fact was known even before but without any phenomenological consequences [15].

Let us suppose now that we have mass M and we know that it hadronizes into $N = \langle n \rangle$ bosonic particles (assumed to be pions of mass m) with equal numbers of (+/-/0) charges and with limited transverse momenta p_T . Let the multiplicity distribution of these pions follows some NB-like form, broader than Poissonian one. Suppose also that the two-particle correlation function of identical particles, $C_2(Q)$, has the specific BEC form mentioned above. How to model such process from the very beginning, i.e., in such way that bosonic character of produced particles is accounted for from the very beginning and not imposed at the end? We propose the following steps (illustrated by comparison to some selected LEP e^+e^- data [16]):

- (1) Using some (assumed) function $f(E)$ select a particle of energy $E_1^{(1)}$ and charge $Q^{(1)}$. The actual form of $f(E)$ should reflect somehow our *a priori* knowledge of the particular collision process under consideration. In what follows we shall assume that $f(E) = \exp(-E/T)$, with T being parameter (playing in our example the role of "temperature").
- (2) Treat this particle as seed of the first *elementary emitting cell* (EEC) and add to it, until the first failure, other particles of the same charge $Q^{(1)}$ selected according to distribution $P(E) = P_0 \cdot f(E)$, where P_0 is another parameter (actually it plays here the role of "chemical potential" $\mu = T \cdot \ln P_0$). This assures that the number of particles in this first EEC, k_1 , will follow geometrical (or Bose-Einstein) distribution, and in precisely this way one accounts for the bosonic character of produced pions. This results in $C_2(Q) > 1$ but only *at one point*, namely for $Q = 0$.
- (3) To get the observed spread out of $C_2(Q)$ one has to allow that particles in this EEC have (slightly) different energies from energy of the particle being its seed. To do it allow that each additional particle selected in point (2) above have energy $E_i^{(1)}$ selected from some distribution function peaked at $E_1^{(1)}$, $G(E_1^{(1)} - E_i^{(1)})$.
- (4) Repeat points (1) to (2) as long as there is enough energy left. Correct in every event for every energy-momentum nonconservation caused by selection

procedure and assure that $N^{(+)} = N^{(-)}$.

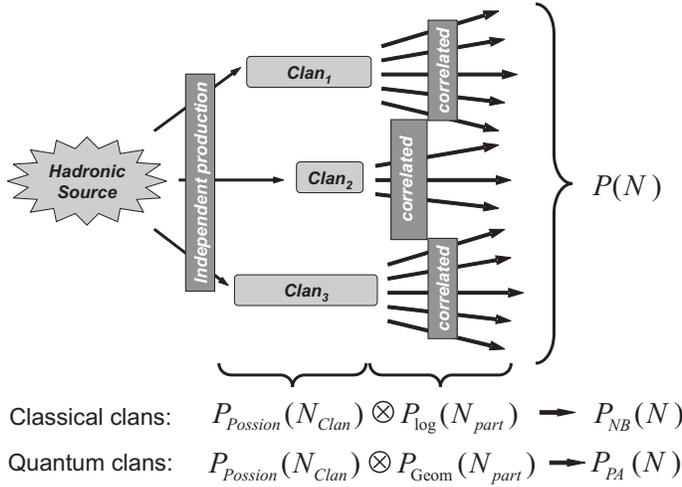


Figure 3: Schematic view of our algorithm, which leads to bunches of particles (*clans*). Whereas in [10] these clans could consist of any particles distributed logarithmically in our case they consist of particles of the same charge and (almost) the same energy and are distributed geometrically to comply with their bosonic character.

As result we get a number of EECs with particles of the same charge and (almost) the same energy, which we regard as being equivalent to *clans* in the [10] (see Fig. 3). These clans are distributed in the same way as the particles forming the seeds for those EEC, i.e., according to Poisson distribution (see Fig. 4, upper-left panel). On the other hand, as was already said, particles in each EEC will be distributed according to geometrical distribution (see Fig. 4, upper-right panel). As a result the overall distribution of particles will be of the so called Pòlya-Aeppli distribution [17]. It fits our examplatory data reasonable well. It is interesting to notice at this point that to get NB distribution resulting from the classical clan model of [10] one should have logarithmic rather than geometrical distribution of particles in EEC, which would then not account for the bosonic character of produced secondaries. In this respect our model differs from this *classical* clan model and we see that what we have obtained is indeed its *quantum* version, therefore its proposed name: *quantum clan model*.

The first preliminary results presented in Fig. 4 are quite encouraging (especially when one remembers that so far effects of resonances and all kind of final state interactions to which C_2 is sensitive were neglected here). It remains now to be checked what two-body BEC functions for other components of the momentum differences and how they depend on the EEC parameters: T , P_0 and σ . So far the main outcome is that BEC are due to EEC's only and therefore provide us mainly with their characteristics (it is worth to mention at this point that essentially this type of approach has been also proposed to simulate Bose-Einstein condensate phenomenon in [18]). This should clear at least some of many apparently

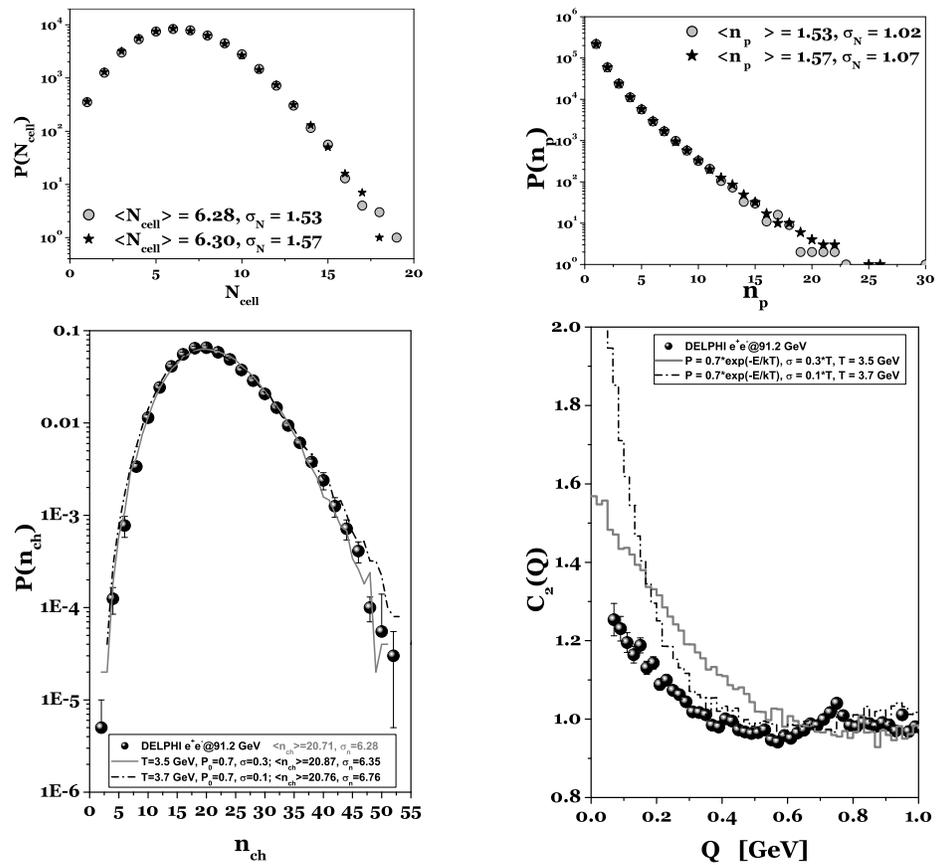


Figure 4: Upper panels: distribution of cells and particles in a given cell. Lower-left panel: the corresponding summary $P(n)$ which is convolution of both $P(n_{\text{cell}})$ and $P(n_p)$. Lower-right panel: examples of the corresponding correlation functions $C_2(Q)$. Two sets of parameters were used. Data are from [16].

"strange" results obtained from BEC recently (see Quark Matter 2004 proceedings, especially [19]). The most intriguing is the fact that apparently the "size" of the hadronizing source deduced from the BEC data does not vary very much with energy and with the size of colliding objects as has been naively expected [2]. In our approach this has simple explanation, see Fig. 5. The point is that BEC are mainly sensitive to the correlation length, which in our case is dimension of the emitting cell, not to dimension of the "fireball" in which hadronization process takes place. The size of this fireball depends mainly on the number of produced secondaries [19], which in our case is given by the "partition temperature" parameter T and by the "chemical potential" parameter $\mu = T \cdot \ln P_0$. They are changing with mass M (and therefore with the energy of reaction and the type of projectile). On the other hand dimension of EEC is given entirely by param-

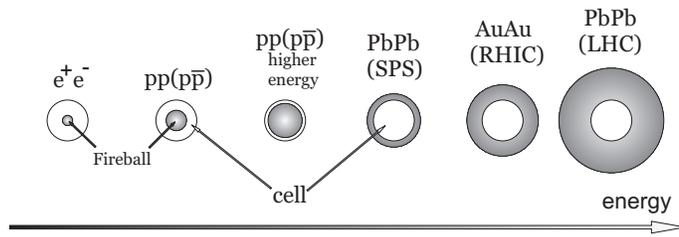


Figure 5: Schematic view of how in our approach the size of the EEC compares with the size of hadronizing fireball at different energies and for different types of projectiles.

eter σ describing the spread of energy of particles belonging to this EEC, which is only weakly depending on energy (if at all). We shall close with mentioning that our approach accounts also for multiparticle BEC and, because of this, by intermittence effects seen in data (at least to some extent) [6].

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