

Meissner masses in the gCFL phase of QCD

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Abstract

We calculate the Meissner masses of gluons in neutral three-flavor color superconducting matter for finite strange quark mass. In the CFL phase the Meissner masses are slowly varying function of the strange quark mass. For large strange quark mass, in the so called gCFL phase, the Meissner masses of gluons with colors $a = 1, 2$ become imaginary, indicating an instability.

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I. INTRODUCTION

At asymptotic densities cold quark matter is in the Color Flavor Locked (CFL) phase of QCD [1, 2]. This state is characterized by nine gapped fermionic quasi-particles (3×3 , for color and flavor) and by electric neutrality even for non vanishing quark masses M_j , provided $M_j \neq 0$ does not destroy the CFL phase [3]. For lower densities, it has recently been shown [4, 5, 6] that, including the strange quark mass M_s , requiring electrical and color neutrality, and imposing weak equilibrium, a phase transition occurs, from the CFL phase to a new phase, called gapless CFL or gCFL. In the gCFL phase seven fermionic quasiparticles have a gap in the dispersion law, but the remaining two are gapless. At zero temperature the transition from the CFL to the gCFL phase takes place at $M_s^2/\mu_b \approx 2\Delta$, where μ_b is the quark chemical potential and Δ is the gap parameter. At non zero temperature the situation is more involved [6] and also the existence of mixed phases [7] has to be taken into account. The next phase at still lower densities is difficult to determine and the crystalline color superconductive phase is a candidate [8].

The aim of this paper is to investigate the dependence of Meissner masses on the strange quark mass in the gCFL phase. For two flavors a similar analysis has recently been performed by Huang and Shovkovy [9]. Imposing weak equilibrium and neutrality they compute the gluon Meissner mass in the 2SC phase and show that an instability arises in a certain range of values of the parameters, with some gluon masses becoming imaginary. We present numerical evidence that a similar instability is also present in the gCFL phase. By the same method we also investigate the dependence of Meissner masses on M_s in the gapped phase (CFL with $M_s \neq 0$). Our calculational scheme is the High Density Effective Theory (HDET) [10, 11], which allows a significant reduction of the computational complexity.

The plan of this paper is as follows. In Section II we derive the effective Lagrangian and the grand potential in the HDET scheme. In Section III we determine the polarization tensor for the gluons in the HDET approximation. In Section IV we present and discuss the numerical results for the Meissner masses as a function of the strange quark mass. The conclusions are summarized in Section V.

II. HDET APPROACH TO THE GCFL PHASE

Following Ref.[5] the Lagrangian for gluons and ungapped quarks with $M_u = M_d = 0$ and $M_s \neq 0$ can be written as follows (color, flavor and spin indices suppressed):

$$\mathcal{L} = \bar{\psi} (i \not{D} - \mathbf{M} + \mu \gamma_0) \psi \quad (1)$$

where $\mathbf{M} = \text{diag}(0, 0, M_s)$ is the mass matrix in flavor space and the matrix of chemical potential is given by [4]

$$\mu_{ij}^{\alpha\beta} = (\mu_b \delta_{ij} - \mu_Q Q_{ij}) \delta^{\alpha\beta} + \delta_{ij} \left(\mu_3 T_3^{\alpha\beta} + \frac{2}{\sqrt{3}} \mu_8 T_8^{\alpha\beta} \right) \quad (2)$$

($i, j = 1, 3$ flavor indices; $\alpha, \beta = 1, 3$ color indices). Moreover $T_3 = \frac{1}{2} \text{diag}(1, -1, 0)$, $T_8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2)$ in color space and $Q = \text{diag}(2/3, -1/3, -1/3)$ in flavor space; μ_Q is the electrostatic chemical potential; μ_3, μ_8 are the color chemical potentials associated respectively to the color charges T_3 and T_8 ; μ_b is quark chemical potential which we fix to 500 MeV. As usual in the HDET, to get rid of the Dirac structure we introduce velocity dependent fields of positive (negative) energy $\psi_{\mathbf{v}}(\Psi_{\mathbf{v}})$ by the Fourier decomposition [11]

$$\psi = \sum_{\mathbf{v}} e^{i \mu_b \mathbf{v} \cdot \mathbf{x}} (\psi_{\mathbf{v}} + \Psi_{\mathbf{v}}), \quad (3)$$

where \mathbf{v} is a unit vector representing the Fermi velocity of the quarks. Substituting the expression (3) in the Eq.(1) at the leading order in M_s^2/μ_b we obtain the HDET Lagrangian

$$\mathcal{L} = \sum_{\mathbf{v}} \psi_{\mathbf{v}}^\dagger \left(iV \cdot D + \delta\mu - \frac{M^2}{2\mu_b} \right) \psi_{\mathbf{v}} - P_{\mu\nu} \psi_{\mathbf{v}}^\dagger \left(\frac{D_\mu D_\nu}{i\tilde{V} \cdot D + 2\mu_b} \right) \psi_{\mathbf{v}}, \quad (4)$$

where $V^\mu = (1, \mathbf{v})$, $\tilde{V}^\mu = (1, -\mathbf{v})$ and

$$P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2} (V^\mu \tilde{V}^\nu + \tilde{V}^\mu V^\nu). \quad (5)$$

It is clear that, at this order of approximation, the effect of $M_s \neq 0$ is to reduce the chemical potential of the strange quarks. Let us now define a new basis ψ_A for the spinor fields:

$$\psi_{\alpha i} = \sum_{A=1}^9 (F_A)_{\alpha i} \psi_A, \quad (6)$$

where the matrices F_A can be expressed by

$$F_1 = \frac{1}{3} I_0 + T_3 + \frac{1}{\sqrt{3}} T_8, \quad F_2 = \frac{1}{3} I_0 - T_3 + \frac{1}{\sqrt{3}} T_8, \quad F_3 = \frac{1}{3} I_0 - \frac{2}{\sqrt{3}} T_8, \quad (7)$$

$$F_{4,5} = T_1 \pm i T_2, \quad F_{6,7} = T_4 \pm i T_5, \quad F_{8,9} = T_6 \pm i T_7, \quad (8)$$

with $T_a = \lambda_a/2$ the $SU(3)$ generators and I_0 the identity matrix. Introducing the Nambu-Gorkov fields

$$\chi_A = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_{\mathbf{v}} \\ C \psi_{-\mathbf{v}}^* \end{pmatrix}_A \quad (9)$$

the kinetic part of the Lagrangian (4) reads

$$\mathcal{L}_0 = \sum_{\mathbf{v}} \chi_A^\dagger \begin{pmatrix} \left(V \cdot \ell + \delta\mu_A - \frac{M_A^2}{2\mu_b} \right) \delta_{AB} & 0 \\ 0 & \left(\tilde{V} \cdot \ell - \delta\mu_A + \frac{M_A^2}{2\mu_b} \right) \delta_{AB} \end{pmatrix} \chi_B \quad (10)$$

where

$$\delta\mu_A = (\delta\mu_{ru}, \delta\mu_{gd}, \delta\mu_{bs}, \delta\mu_{rd}, \delta\mu_{du}, \delta\mu_{rs}, \delta\mu_{bu}, \delta\mu_{gs}, \delta\mu_{bd}) \quad (11)$$

and $M_A^2 = m_s^2(0, 0, 1, 0, 0, 1, 0, 1, 0)$. If we define

$$\delta\mu_A^{\text{eff}} = \delta\mu_A - \frac{M_A^2}{2\mu_b}, \quad (12)$$

we may recast Eq. (10) as

$$\mathcal{L}_0 = \sum_{\mathbf{v}} \chi_A^\dagger \begin{pmatrix} \left(V \cdot \ell + \delta\mu_A^{\text{eff}} \right) \delta_{AB} & 0 \\ 0 & \left(\tilde{V} \cdot \ell - \delta\mu_A^{\text{eff}} \right) \delta_{AB} \end{pmatrix} \chi_B, \quad (13)$$

which is formally equivalent to the Lagrangian for massless quarks with different chemical potentials.

In the gapless color-flavor-locking (gCFL) phase the symmetry breaking is induced by the condensate [4]

$$\Delta_{ij}^{\alpha\beta} \equiv \langle \psi_{i\alpha} C \gamma_5 \psi_{\beta j} \rangle = \sum_{I=1}^3 \Delta_I \epsilon^{\alpha\beta I} \epsilon_{ijI} \quad (14)$$

and the corresponding gap term in the Lagrangian in the mean field approximation is

$$\mathcal{L}_\Delta = -\frac{1}{2} \Delta_{ij}^{\alpha\beta} \sum_{\mathbf{v}} \psi_{\alpha i, -\mathbf{v}}^T C \gamma_5 \psi_{j\beta, +\mathbf{v}} + h.c.. \quad (15)$$

In the Nambu-Gorkov basis (9) the gap term reads

$$\mathcal{L}_\Delta = \sum_{\mathbf{v}} \chi_A^\dagger \begin{pmatrix} 0 & -\Delta_{AB} \\ -\Delta_{AB} & 0 \end{pmatrix} \chi_B, \quad (16)$$

where Δ_{AB} is the 9×9 matrix defined by

$$\Delta_{AB} = -\sum_{I=1}^3 \Delta_I \text{Tr} [F_A^T \epsilon_I F_B \epsilon_I]$$

$$= \begin{pmatrix} 0 & \Delta_3 & \Delta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta_3 & 0 & \Delta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta_2 & \Delta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Delta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Delta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\Delta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_1 & 0 \end{pmatrix}. \quad (17)$$

From Eqs.(13) and (16) one immediately obtains the inverse fermion propagator that in momentum space is given by

$$S_{AB}^{-1}(\ell) = \begin{pmatrix} (V \cdot \ell + \delta\mu_A^{eff})\delta_{AB} & -\Delta_{AB} \\ -\Delta_{AB} & (\tilde{V} \cdot \ell - \delta\mu_A^{eff})\delta_{AB} \end{pmatrix}. \quad (18)$$

It can be inverted to get the fermion propagator

$$S_{AB}(l) = \begin{pmatrix} (P \Delta (\tilde{V} \cdot l - \delta\mu_{eff}) \Delta^{-1})_{AB} & P_{AC} \Delta_{CB} \\ D_{AC} \Delta_{CB} & (D \Delta (V \cdot l + \delta\mu_{eff}) \Delta^{-1})_{AB} \end{pmatrix} \quad (19)$$

where

$$P = \frac{1}{\Delta (\tilde{V} \cdot l - \delta\mu_{eff}) \Delta^{-1} (V \cdot l + \delta\mu_{eff}) - \Delta^2} \quad (20)$$

and $D = P(V \leftrightarrow \tilde{V}, \delta\mu_{eff} \leftrightarrow -\delta\mu_{eff})$. From the poles of the propagator we can now determine the dispersion laws of the quasiparticles. The knowledge of the dispersion laws allows the evaluation of the grand potential which in the limit of zero temperature is given by

$$\Omega = -\frac{1}{2\pi^2} \int_0^\Lambda dp p^2 \sum_{j=1}^9 |\epsilon_j(\ell_{\parallel})| + \frac{1}{G} (\Delta_1^2 + \Delta_2^2 + \Delta_3^2) - \frac{\mu_Q^4}{12\pi^2}, \quad (21)$$

where Λ is the ultraviolet cutoff, $\epsilon_j(\ell_{\parallel})$ are the quasi particle dispersion laws, ℓ_{\parallel} is the quark momentum measured from the Fermi surface ($p = \mu_b + \ell_{\parallel}$), and G is the Nambu-Jona Lasinio coupling constant. We fix G as in [4], requiring that the value of the gap is 25 Mev for $M_s = 0$. In order to enforce electrical and color neutrality one has to minimize the grand potential with respect to μ_Q, μ_3 and μ_8 . Including the stationary conditions with respect to the gap parameters $\Delta_1, \Delta_2, \Delta_3$ one ends up with a system of six equations which must be

solved simultaneously. Once this system of equations is solved one may express $\Delta_1, \Delta_2, \Delta_3$ and the chemical potentials μ_Q, μ_3 and μ_8 as a function of M_s^2/μ_b . We have numerically checked that using the grand potential (21), with $\mu_b = 500 \text{ MeV}$ and $\Lambda = 800 \text{ MeV}$, we recover the results of Ref. [4] with an error of 5%.

III. POLARIZATION TENSOR OF GLUONS

To compute gluon Meissner masses we evaluate the polarization tensor $\Pi_{ab}^{\mu\nu}(p)$. In the HDET approach, at the leading order in $g\mu_b$, there are two contribution to the polarization tensor: The self-energy (s.e.) diagram and the tadpole (tad) diagram (see e.g. Fig. 2 in [12]). To evaluate the self-energy diagram we extract the trilinear quark-gluon coupling by the minimal coupling term in the Lagrangian (4):

$$\mathcal{L}_1 = i g \sum_{\mathbf{v}} \psi_{i\alpha, \mathbf{v}}^\dagger i V^\mu A_\mu^a (T_a)^{\alpha\beta} \psi_{\beta j, \mathbf{v}} \quad (22)$$

which can be rewritten in the Nambu-Gorkov basis (9) as

$$\mathcal{L}_1 = i g \sum_{\mathbf{v}} \chi_A^\dagger \begin{pmatrix} i V \cdot A_a h_{AB}^a & 0 \\ 0 & -i \tilde{V} \cdot A_a h_{AB}^{a*} \end{pmatrix} \chi_B \equiv i g \sum_{\mathbf{v}} \chi_A^\dagger \tilde{H}_{AB}^{a\mu} \chi_B A_\mu^a \quad (23)$$

where $h_{AB}^a = \text{Tr}[F_A^\dagger T^a F_B]$. Therefore the self-energy contribution to the polarization tensor is given by:

$$i \Pi_{ab}^{s.e., \mu\nu}(p) = \frac{g^2 \mu_b^2}{4 \pi^3} \int \frac{d\mathbf{v}}{4 \pi} \int d^2 \ell \text{Tr} [S(\ell) \tilde{H}^{a\mu} S(\ell + p) \tilde{H}^{b\nu}] . \quad (24)$$

In order to evaluate the tadpole diagram contribution we extract the quadrilinear quark-gluon coupling from the second term on the r.h. side of Eq.(4)

$$\mathcal{L}_2 = -g^2 \sum_{\mathbf{v}} \psi_{\mathbf{v}}^\dagger \frac{T_a T_b}{\tilde{V} \cdot \ell + 2 \mu_b} \psi_{\mathbf{v}} P_{\mu\nu} A_a^\mu A_b^\nu . \quad (25)$$

In the Nambu-Gorkov basis this term reads

$$\mathcal{L}_2 = -g^2 \sum_{\mathbf{v}} \chi_A^\dagger \begin{pmatrix} \frac{d_{AB}^{ab}}{\tilde{V} \cdot \ell + 2 \mu_b} & 0 \\ 0 & \frac{d_{AB}^{a*b}}{\tilde{V} \cdot \ell + 2 \mu_b} \end{pmatrix} \chi_B P_{\mu\nu} A_a^\mu A_b^\nu \equiv -g^2 \sum_{\mathbf{v}} \chi_A^\dagger Y_{AB}^{ab} \chi_B P_{\mu\nu} A_a^\mu A_b^\nu , \quad (26)$$

with $d_{AB}^{ab} = \text{Tr}[F_A^\dagger T^a T^b F_B]$. The tadpole contribution is then evaluated to be

$$i \Pi_{ab}^{tad, \mu\nu} = -2 g^2 \frac{4 \pi}{16 \pi^4} \int \frac{d\mathbf{v}}{4 \pi} P^{\mu\nu} \int d\ell d\ell_\parallel \text{Tr} \left[i S(\ell) \begin{pmatrix} (\mu_b + \ell_\parallel)^2 & 0 \\ 0 & (\mu_b - \ell_\parallel)^2 \end{pmatrix} Y^{ab} \right] . \quad (27)$$

Finally, in the HDET approximation, the gluon polarization tensor is given by:

$$\Pi_{ab}^{\mu\nu}(p) = \Pi_{ab}^{s.e.,\mu\nu}(p) + \Pi_{ab}^{tad,\mu\nu}(p). \quad (28)$$

We note that this result for the polarization tensor is correct at the order $\mathcal{O}(M_s/\mu_b)^2$.

IV. NUMERICAL RESULTS

The Meissner masses of the gluons are obtained by the eigenvalues of the polarization tensor (28) in the static limit $p_0 = 0, \mathbf{p} \rightarrow 0$. In the CFL phase with $M_s = 0$ the Meissner masses are degenerate and one has [13, 14]

$$m_M^2 = \frac{\mu_b^2 g^2}{\pi^2} \left(-\frac{11}{36} - \frac{2}{27} \ln 2 + \frac{1}{2} \right). \quad (29)$$

For a non zero strange quark mass the integrals in Eqs. (24) and (27) have to be evaluated numerically. In Fig. 1 we present the results for the squared Meissner masses of gluons with color $a = 1, 2, 3, 8$ in units of m_M^2 . The solid line denotes gluons with color $a = 1, 2$, the short-dashed line gluons with color $a = 3$ and the long-dashed line gluons with $a = 8$. We find that increasing M_s^2/μ_b the degeneracy in the Meissner masses is partially removed. Moreover there is a discontinuity of the squared Meissner mass of gluons of colors $a = 1, 2$, which at the onset of the gCFL phase, i.e. for $M_s^2/\mu_b \sim 2\Delta$, drop to negative values. Thus we find an instability in the gCFL phase analogous to the one observed by Huang and Shovkovy [9] in the g2SC case. On the other hand we note that gluons of colors $a = 3$ and $a = 8$ are continuous positive functions of M_s^2/μ_b both in the CFL and gCFL phase.

In Fig. 2 we present the results for the gluons with color $a = 4, 5$ (solid line) and color $a = 6, 7$ (dashed line). Also in these cases the squared Meissner mass of gluons are continuous functions of M_s^2/μ_b . One can notice that, for very large values of the strange quark mass, the squared Meissner masses of these gluons become negative. However this result is not robust because in the computation of the polarization tensor Eq.(28) we have discarded terms of order $\mathcal{O}(M_s/\mu_b)^2$. Therefore, to establish the instability related to the gluons $a = 4, 5, 6$ and 7 a more accurate analysis would be needed. Also in this case, as with previous Fig. 1, our results give not only the Meissner masses in the gCFL phase, but also their dependence on the strange quark mass in the CFL phase, i.e. for $M_s^2/\mu_b \leq 2\Delta$.

The instability we have found means that the vacuum was not correctly identified. A possible origin of the instability is a non vanishing vacuum expectation value (vev) of one (or

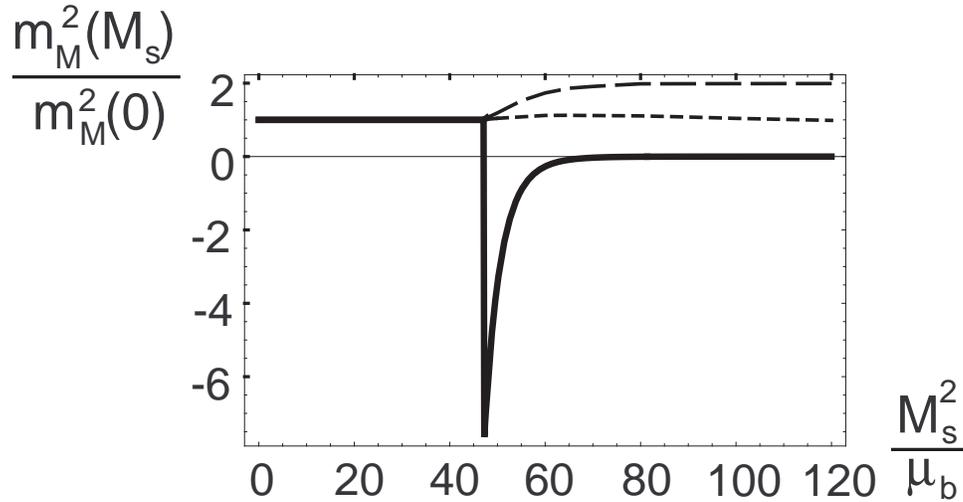


FIG. 1: Squared values of the Meissner masses in units of m_M^2 (see Eq. (29)) as a function of M_s^2/μ_b (in MeV) for gluons $a = 1, 2, 3, 8$. The solid line denotes the gluons with colors $a = 1, 2$. The short-dashed line denotes the gluons with color $a = 3$ and the long-dashed line $a = 8$.

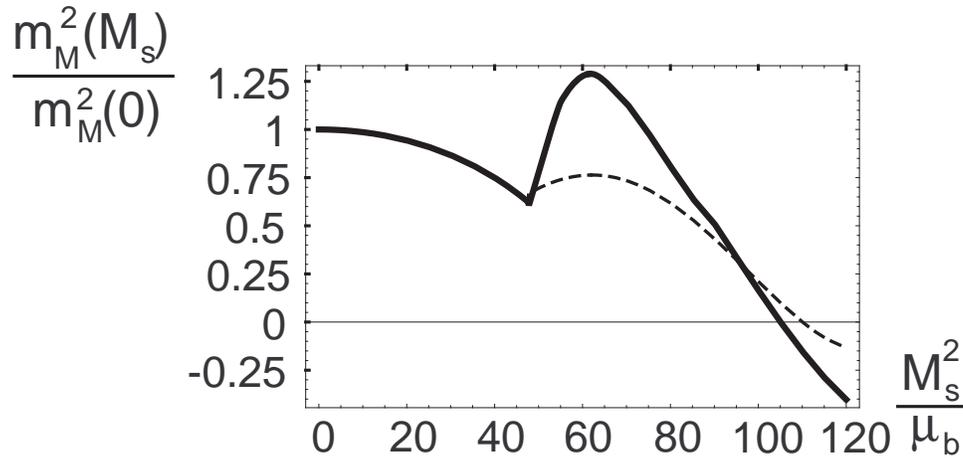


FIG. 2: Squared values of the Meissner masses in units of m_M^2 as a function of M_s^2/μ_b . Dashed line denotes the gluons with colors $a = 4, 5$; solid line the gluons with colors $a = 6, 7$.

more) time components of the gluon operator A_a^μ : $\langle A_a^0 \rangle \neq 0$. Clearly defining a new field operator with vanishing vev's adds contributions that, for $a = 3, 8$, act as effective chemical potential terms in the lagrangian: $\sim g \langle A_a^0 \rangle \psi^\dagger \lambda_a \psi$. The presence of these new terms would alter previous results and may lead to real Meissner masses. We have numerically

checked that, either with $\langle A_3^0 \rangle \neq 0$ and the other vev's $\langle A_a^0 \rangle = 0$, or with $\langle A_3^0 \rangle \neq 0$ and the other vev's equal to zero, one removes the instability (numerically the non vanishing vev's must be of the order of ~ 10 MeV). At present the physical mechanism at the basis of this gluon condensation is still unclear.

V. CONCLUSIONS

It is well established that at asymptotic large densities quark matter is in the CFL phase. At lower densities, in a range presumably more relevant for the study of compact stars, neutrality, together with finite strange quark mass, suggests the gCFL (gapless CFL) phase as the next occurring ground state. Our calculations in this note suggest an instability of the gCFL phase, a phenomenon analogous to what observed in the two flavor case. The instability arises because gluons of color indices 1 and 2 present an imaginary mass. Its removal may require a different condensation pattern, most probably including gluon condensation.

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