ANALYSIS OF SO(2N) COUPLINGS OF SPINOR AND TENSOR REPRESENTATIONS IN $SU(N) \times U(1)$ INVARIANT FORMS

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A review is given of a recently developed technique for the analysis of $SO(2N)$ invariant couplings which allows a full exhibition of the $SU(N)$ invariant content of couplings involving the $SO(2N)$ semi-spinors $|\Psi_\pm>$ with chiralilty $\pm$ and tensor representations. We discuss the Basic Theorem used in the analysis and then exhibit the technique by illustrative examples for the computation of the trilinear and quartic couplings for the $SO(10)$ case involving three generations of 16 plets of matter.

1. Introduction

In this paper we give a brief overview of a recently developed technique for the computation of $SO(2N)$ couplings of spinor and tensor representations in $SU(N) \times U(1)$ invariant forms. These techniques are then specifically applied in illustrative examples for the computation of $SO(10)$ invariant couplings when they are decomposed in $SU(5) \times U(1)$ invariant forms. The analysis presented here is of relevance in view of the importance of $SO(10)$ as a grand unification group[1] of the electroweak and the strong interactions. The techniques used are based on the oscillator method[2,3,4] and developed further in Refs [6,7,8] while some related work can be found in Ref.[5]. This paper is thus essentially a brief summary of the works of Refs.[6,7]. The outline of the rest of the paper is as follows: In Sec.2 we give a brief discussion of the $SO(2N)$ algebra, $SO(2N)$ spinor representations for N odd, form of $SO(2N)$ invariant couplings, and specialization to the $SO(10)$ case. In Sec.3 we give a brief review of the new technique for the evaluation of the $SO(2N)$ invariant couplings in terms of $SU(N) \times U(1)$ invariant forms. In this section we also discuss the Basic Theorem derived in Ref.[6]. In Sec.4 we specialize to the $SO(10)$ case and give illustrative examples of the computation of cubic couplings in the superpotential and in the Lagrangian. These involve couplings $16 - 16 - \overline{126}$ in the superpotential, and the couplings of 16 plets with 45 of gauge fields, i.e., the couplings $16-16-45$, in the Lagrangian. . We also discuss a sample of $SO(10)$ invariant quartic couplings.

2. Spinor representations of $SO(2N)$; N odd

In this section we will discuss the embedding of $SU(N)$ in $SO(2N)$. Further, we list the general expressions for the group invariants formed from different combinations of $SO(2N)$ spinors. Finally, we specialize the results to $SO(10)$ grand unification group.

2.1. $SO(2N)$ *algebra in a basis*

The 2^N dimensional spinor, $|\Psi>$ of $SO(2N)$ splits into two inequivalent 2^{N-1} dimensional semi-spinors, $|\Psi_{(\pm)}>$ under the action of the chirality operator: $|\Psi_{(\pm)}>=\frac{1}{2}(1\pm\Gamma_0)|\Psi>$ where $\Gamma_0=i^N\Gamma_1\Gamma_2...\Gamma_{2N}$ and further $\Gamma_0|\Psi_{(\pm)}>=\pm|\Psi_{(\pm)}>$. Here Γ_μ ($\mu=1,2,..,2N$) define a rank $2N$ Clifford algebra with $\{\Gamma_\mu,\Gamma_\nu\}=2\delta_{\mu\nu}$, and $\Sigma_{\mu\nu}=\frac{1}{2i}[\Gamma_\mu,\Gamma_\nu]$ are the $N(2N-1)$ generators of $SO(2N)$. Further, it is convenient to introduce operators b_i and $b_i^\dagger$ such that $\{b_i,b_j^\dagger\}=\delta_{ij}$, $\{b_i,b_j\}=0=\{b_i^\dagger,b_j^\dagger\}$. The semi-spinors, $|\Psi_{(\pm)}>$ of $SO(2N)$ can be expanded in terms of reducible antisymmetric $SU(N)$ tensors M, N as follows

$$|\Psi_{(+)}>=\sum_{p=0,2,..}^{N-1}\frac{1}{p!}\mathcal{M}_{(p)}^{i_1...i_p}\prod_{q=2,4,..}^{p}b_{i_q}^\dagger|0>,$$

$$|\Psi_{(-)}>=\sum_{p=1,3,..}^{N}\frac{1}{p!}\mathcal{N}_{(p)}^{i_1...i_p}\prod_{q=1,3,..}^{p}b_{i_q}^\dagger|0> \tag{1}$$

where the p-index tensors can be reduced to $(N-p)$-index tensors as

$$\mathcal{M}_{(N-p)i_N...i_{p+1}}=\frac{1}{p!}\epsilon_{i_N...i_1}\mathcal{M}_{(p)}^{i_1...i_p}$$

$$\mathcal{M}_{(p)}^{i_1...i_p\dagger}=\mathcal{M}^*_{(p)i_p...i_1} \tag{2}$$

2.2. $SO(2N)$ *invariant couplings*

Couplings formed from $\Psi^\dagger$ and Ψ are given by

$$\mathsf{g}_{ab}<\Psi_{(\pm)a}|\Gamma_{[\mu_1}..\Gamma_{\mu_p]}|\Psi_{(\mp)b}>\Phi_{\mu_1..\mu_p},\ p=1,3,..,N \tag{3}$$

$$\mathsf{g}_{ab}<\Psi_{(\pm)a}|\Gamma_{[\mu_1}..\Gamma_{\mu_p]}|\Psi_{(\pm)b}>\Phi_{\mu_1..\mu_p},\ p=0,2,..,N-1 \tag{4}$$

where $\mathtt{g}_{ab}$ is a coupling constant where a and b are family indices. $\Phi_{\mu_1\mu_2...\mu_p}$ is a real antisymmetric tensor of $SO(2N)$ with dimensionality $\binom{2N}{p}$. For $p = N$

$$\Phi_{\mu_1..\mu_N} = \overline{\Delta}_{\mu_1..\mu_N} + \Delta_{\mu_1..\mu_N},$$
$$\begin{pmatrix} \overline{\Delta}_{\mu_1..\mu_N} \\ \Delta_{\mu_1..\mu_N} \end{pmatrix} = \pm \frac{i}{N!} \epsilon_{\mu_1..\mu_N \nu_1..\nu_N} \begin{pmatrix} \overline{\Delta}_{\nu_1..\nu_N} \\ \Delta_{\nu_1..\nu_N} \end{pmatrix}. \tag{5}$$

Both $\overline{\Delta}$ and Δ have dimensionality $\frac{1}{2}\binom{2N}{N}$. The symmetry factor in the exchange of identical $\Psi_{(.)}$ is $\mathtt{g}_{ab} = (-1)^{\frac{1}{2}p(p-1)}\mathtt{g}_{ba}$. Couplings formed from $\Psi^{\mathbf{T}}$ and Ψ are given by

$$\mathtt{f}_{ab} < \Psi^*_{(\pm)a} | B\Gamma_{[\mu_1}..\Gamma_{\mu_p]} | \Psi_{(\pm)b>} | \Psi_{(\mp)b} > \Phi_{\mu_1..\mu_p},\ p = 1, 3, .., N \tag{6}$$

$$\mathtt{f}_{ab} < \Psi^*_{(\pm)a} | B\Gamma_{[\mu_1}..\Gamma_{\mu_p]} | \Psi_{(\mp)b} > \Phi_{\mu_1..\mu_p},\ p = 0, 2, .., N-1 \tag{7}$$

where $B = \prod_{\mu=1}^{N} \Gamma_{2\mu-1}$ is the $SO(2N)$ charge conjugation operator and satisfies the relation $\acute{\Sigma}^{\mathbf{T}}_{\mu\nu}\acute{B} = -\acute{\Sigma}_{\mu\nu}\acute{B}$ where $\acute{}$ indicates a $2^N \times 2^N$ matrix representation. The symmetry factor $\mathtt{f}_{ab} = (-1)^{\frac{1}{2}(N-p)(N-p-1)}\mathtt{f}_{ba}$.

2.3. *Specialization to SO(10) gauge group*

We consider now the special case of $SO(10)$ where $\Psi_{(+)} \sim 16, \Psi_{(-)} \sim \overline{16}$ under $SO(10) \supset SU(5) \otimes U(1)$. Here $16 \supset [1] \oplus [\overline{5}] \oplus [10]$, $\overline{16} \supset [1] \oplus [5] \oplus [\overline{10}]$ and $16 \otimes 16 = 10_s \oplus 120_{as} \oplus 126_s$ while $16 \otimes \overline{16} = 1 \oplus 45 \oplus 210$. In terms of their oscillator modes

$$|\Psi_{(+)a}>= |0> \mathcal{M}_a + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \mathcal{M}_a^{ij} + \frac{1}{24} \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathcal{M}_{ai}$$
$$|\Psi_{(-)b}>= \frac{1}{12} \epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathcal{N}_{bij} + b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger |0> \mathcal{N}_b + b_i^\dagger |0> \mathcal{N}_b^i \tag{8}$$

where

$$\mathcal{M}_a = \nu^c_{La},\ \mathcal{M}_{a\alpha} = D^c_{La\alpha},\ \mathcal{M}_a^{\alpha\beta} = \epsilon^{\alpha\beta\gamma} U^c_{La\gamma},\ \mathcal{M}_{a4} = E^-_{La},$$
$$\mathcal{M}_a^{4\alpha} = U_{La\alpha},\ \mathcal{M}_{a5} = \nu_{La},\ \mathcal{M}_a^{45} = E^+_{La},\ \mathcal{M}_a^{5\alpha} = D_{La\alpha} \tag{9}$$

and where α, β, γ are the color indices. Cubic couplings in the superpotential involving $\Psi_{(\pm)}$ are

$$
\begin{aligned}
\mathsf{W}^{(1)} &= f_{ab}^{(1)} < \widehat{\Psi}^*_{(\pm)a}|B|\widehat{\Psi}_{(\mp)b} > \Phi \\
\mathsf{W}^{(45)} &= \frac{1}{2!} f_{ab}^{(45)} < \widehat{\Psi}^*_{(\pm)a}|B\mathbf{\Sigma}_{\mu\nu}|\widehat{\Psi}_{(\mp)b} > \Phi_{\mu\nu} \\
\mathsf{W}^{(210)} &= \frac{1}{4!} f_{ab}^{(210)} < \widehat{\Psi}^*_{(\pm)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}|\widehat{\Psi}_{(\mp)b} > \Phi_{\mu\nu\rho\lambda} \\
\mathsf{W}^{(10)} &= f_{ab}^{(10)} < \widehat{\Psi}^*_{(\pm)a}|B\Gamma_\mu|\widehat{\Psi}_{(\pm)b} > \Phi_\mu \\
\mathsf{W}^{(120)} &= \frac{1}{3!} f_{ab}^{(120)} < \widehat{\Psi}^*_{(\pm)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_{\lambda]}|\widehat{\Psi}_{(\pm)b} > \Phi_{\mu\nu\lambda} \\
\mathsf{W}^{(126,\overline{126})} &= \frac{1}{5!} f_{ab}^{(126,\overline{126})} < \widehat{\Psi}^*_{(\pm)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_\sigma\Gamma_{\lambda]}|\widehat{\Psi}_{(\pm)b} > \begin{pmatrix} \overline{\mathbf{\Delta}}_{\mu\nu\rho\sigma\lambda} \\ \mathbf{\Delta}_{\mu\nu\rho\sigma\lambda} \end{pmatrix}
\end{aligned}
\tag{10}
$$

The semi-spinor $\Psi_{(\pm)}$ with $\widehat{\ }$ stands for a chiral superfield and $SO(10)$ charge conjugation operator is $B = -i\prod_{k=1}^{5}(b_k - b_k^\dagger)$. Couplings in the Lagrangian have the form

$$
\begin{aligned}
\mathsf{L}^{(1)} &= g_{ab}^{(1)} < \Psi_{(\pm)a}|\gamma^0\gamma^A|\Psi_{(\pm)b} > \Phi_A \\
\mathsf{L}^{(45)} &= \frac{1}{2!} g_{ab}^{(45)} < \Psi_{(\pm)a}|\gamma^0\gamma^A\mathbf{\Sigma}_{\mu\nu}|\Psi_{(\pm)b} > \Phi_{A\mu\nu} \\
\mathsf{L}^{(210)} &= \frac{1}{4!} g_{ab}^{(210)} < \Psi_{(\pm)a}|\gamma^0\gamma^A\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}|\Psi_{(\pm)b} > \Phi_{A\mu\nu\rho\lambda}
\end{aligned}
\tag{11}
$$

where A stands for the Lorentz index.

3. Technique for Evaluation of $SO(2N)$ invariants

Here we review the recently developed technique[6,7] for the analysis of $SO(2N)$ invariant couplings which allows a full exhibition of the $SU(N)$ invariant content of the spinor and tensor representations. The technique utilizes a basis consisting of a specific set of reducible $SU(N)$ tensors in terms of which the SO(2N) invariant couplings have a simple expansion.

3.1. *Specific set of $SU(N)$ reducible tensors*

We begin with the observation that the natural basis for the expansion of the $SO(2N)$ vertex is in terms of a specific set of $SU(N)$ reducible tensors, Φ_{c_k} and $\Phi_{\overline{c}_k}$ which we define as $A^k \equiv \Phi_{c_k} \equiv \Phi_{2k} + i\Phi_{2k-1}$, $A_k \equiv \Phi_{\overline{c}_k} \equiv \Phi_{2k} - i\Phi_{2k-1}$. This can be extended immediately to define the quantity $\Phi_{c_i c_j \overline{c}_k ..}$ with an arbitrary number of unbarred and barred indices where each c index can be expanded out so that $A^i A^j A_k ... = \Phi_{c_i c_j \overline{c}_k ...} =$

$\Phi_{2ic_j\overline{c}_k...} + i\Phi_{2i-1c_j\overline{c}_k...}$ etc.. Thus, for example, the quantity $\Phi_{c_ic_j\overline{c}_k...c_N}$ is a sum of 2^N terms gotten by expanding all the c indices. $\Phi_{c_ic_j\overline{c}_k...c_n}$ is completely anti-symmetric in the interchange of its c indices whether unbarred or barred: $\Phi_{c_i\overline{c}_jc_k...\overline{c}_n} = -\Phi_{c_k\overline{c}_jc_i...\overline{c}_n}$. Further, $\Phi^*_{c_i\overline{c}_jc_k...\overline{c}_n} = \Phi_{\overline{c}_ic_j\overline{c}_k...c_n}$ etc.. We now make the observation[6] that the object $\Phi_{c_ic_j\overline{c}_k...c_n}$ transforms like a reducible representation of SU(N). Thus if we are able to compute the SO(2N) invariant couplings in terms of these reducible tensors of SU(N) then there remains only the further step of decomposing the reducible tensors into their irreducible parts.

3.2. *Basic Theorem to evaluate an* $SO(2N)$ *vertex*

A result essential to our analysis is the Basic Theorem[6] which states that an $SO(2N)$ vertex $\Gamma_\mu\Gamma_\nu\Gamma_\lambda...\Gamma_\sigma\Phi_{\mu\nu\lambda...\sigma}$ can be expanded in the following fashion

$$\begin{aligned}\Gamma_\mu\Gamma_\nu\Gamma_\lambda...\Gamma_\sigma\Phi_{\mu\nu\lambda...\sigma} &= b_i^\dagger b_j^\dagger b_k^\dagger...b_n^\dagger\Phi_{c_ic_jc_k...c_n}\\ +(b_ib_j^\dagger b_k^\dagger...b_n^\dagger\Phi_{\overline{c}_ic_jc_k...c_n} + \text{perms}) &+ (b_ib_jb_k^\dagger...b_n^\dagger\Phi_{\overline{c}_i\overline{c}_jc_k...c_n} + \text{ perms}) + ...\\ &+(b_ib_jb_k...b_{n-1}b_n^\dagger\Phi_{\overline{c}_i\overline{c}_j\overline{c}_k...\overline{c}_{n-1}c_n} + \text{ perms})\\ &+b_ib_jb_k...b_n\Phi_{\overline{c}_i\overline{c}_j\overline{c}_k...\overline{c}_n} \qquad (12)\end{aligned}$$

The result of Eq.(12) is very useful in the computation of $SO(10)$ invariant couplings.

4. Cubic Couplings of SO(10)

In this section we give illustrative examples of some $SO(10)$ trilinear couplings in their $SU(5)$ decomposed form. These illustrative examples consist of $16-16-45$ couplings in the Lagrangian and the $16-16-\overline{126}$ coupling in the superpotential.

4.1. $\mathbf{16}\otimes\mathbf{16}\otimes\mathbf{45}$ *coupling in the Lagrangian*

The interaction Lagrangian of the 45 of gauge fields with the 16-plet of $SO(10)$ spinor $|\Psi_{(+)}>$ is given by

$$\mathsf{L}^{(45)} = \frac{1}{i}\frac{1}{2!}g_{ab}^{(45)} < \Psi_{(+)a}|\gamma^0\gamma^A\Sigma_{\mu\nu}|\Psi_{(+)b} > \Phi_{A\mu\nu}. \qquad (13)$$

Expansion of the vertex gives

$$\Sigma_{\mu\nu}\Phi_{\mu\nu} = \frac{1}{i}(b_ib_j\Phi_{\overline{c}_i\overline{c}_j} + b_i^\dagger b_j^\dagger\Phi_{c_ic_j} + 2b_i^\dagger b_j\Phi_{c_i\overline{c}_j} - \Phi_{c_n\overline{c}_n}). \qquad (14)$$

The 45 of $SO(10)$ decomposes under $SU(5)$ as $45 \supset 1(\mathsf{g}) \oplus 10(\mathsf{g}^{ij}) \oplus \overline{10}(\mathsf{g}_{ij}) \oplus \overline{24}(\mathsf{g}^i_j)$ where

$$\Phi_{c_n\overline{c}_n} = \mathsf{g}, \quad \Phi_{c_i\overline{c}_j} = \mathsf{g}^i_j + \frac{1}{5}\delta^i_j\mathsf{g}$$
$$\Phi_{c_ic_j} = \mathsf{g}^{ij}, \quad \Phi_{\overline{c}_i\overline{c}_j} = \mathsf{g}_{ij}. \tag{15}$$

The normalized $SU(5)$ gauge fields are

$$\mathsf{g}_A = 2\sqrt{5}\mathsf{G}_A, \quad \mathsf{g}_{Aij} = \sqrt{2}\mathsf{G}_{Aij}$$
$$\mathsf{g}^{ij}_A = \sqrt{2}\mathsf{G}^{ij}_A, \quad \mathsf{g}^i_{Aj} = \sqrt{2}\mathsf{G}^i_{Aj} \tag{16}$$

In terms of the redefined fields, the kinetic energy of the 45-plet takes the form

$$-\frac{1}{4}F^{AB}_{\mu\nu}F_{AB\mu\nu} = -\frac{1}{2}\ G_{AB}G^{AB\dagger} - \frac{1}{2!}\frac{1}{2}G^{ABij}G^{ij\dagger}_{AB} - \frac{1}{4}G^{ABi}_jG^j_{ABi} \tag{17}$$

where $F^{AB}_{\mu\nu}$ is the 45 of SO(10) field strength tensor

$$\begin{aligned}\mathsf{L}^{(45)} = g^{(45)}_{ab}[\sqrt{5}(&-\frac{3}{5}\overline{\mathcal{M}}^i_a\gamma^A\mathcal{M}_{bi} + \frac{1}{10}\overline{\mathcal{M}}_{aij}\gamma^A\mathcal{M}^{ij}_b + \overline{\mathcal{M}}_a\gamma^A\mathcal{M}_b)\mathsf{G}_A\\
&+\frac{1}{\sqrt{2}}(\overline{\mathcal{M}}_a\gamma^A\mathcal{M}^{lm}_b + \frac{1}{2}\epsilon^{ijklm}\overline{\mathcal{M}}_{aij}\gamma^A\mathcal{M}_{bk})\mathsf{G}_{Alm}\\
&-\frac{1}{\sqrt{2}}(\overline{\mathcal{M}}_{alm}\gamma^A\mathcal{M}_b + \frac{1}{2}\epsilon_{ijklm}\overline{\mathcal{M}}^i_a\gamma^A\mathcal{M}^{jk}_b)\mathsf{G}^{lm}_A\\
&+\sqrt{2}(\overline{\mathcal{M}}_{aik}\gamma^A\mathcal{M}^{kj}_b + \overline{\mathcal{M}}^j_a\gamma^A\mathcal{M}_{bi})\mathsf{G}^i_{Aj}]\end{aligned} \tag{18}$$

The barred matter fields are defined so that $\overline{\mathcal{M}}_{ij} = \mathcal{M}^\dagger_{ij}\gamma^0$.

4.2. $\mathbf{16 \otimes 16 \otimes \overline{126}}$ *coupling in the superpotential*

The $\overline{126}$ of $SO(10)$ decomposes under $SU(5)$ as $\overline{126} \supset 1(\mathsf{H}) \oplus 5(\mathsf{H}^i) \oplus \overline{10}(\mathsf{H}_{ij}) \oplus 15(\mathsf{H}^{ij}_{(S)}) \oplus \overline{45}(\mathsf{H}^k_{ij}) \oplus 50(\mathsf{H}^{ij}_{kl})$. Utilizing the Basic Theorem the result for the Yukawa coupling involving $\overline{126}$ of Higgs is as follows

$$\begin{aligned}\mathsf{W}^{(\overline{126})} &= \frac{1}{5!}f^{(\overline{126})}_{ab} < \widehat{\Psi}^*_{(+)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_\lambda\Gamma_{\sigma]}|\widehat{\Psi}_{(+)b} > \overline{\Delta}_{\mu\nu\rho\lambda\sigma}\\
&= i\sqrt{\frac{2}{15}}f^{(\overline{126})(+)}_{ab}[-\sqrt{2}\widehat{\mathcal{M}}^{\mathbf{T}}_a\widehat{\mathcal{M}}_b\mathsf{H} + \widehat{\mathcal{M}}^{\mathbf{T}}_a\widehat{\mathcal{M}}^{ij}_b\mathsf{H}_{ij}\\
&\quad -\widehat{\mathcal{M}}^{\mathbf{T}}_{ai}\widehat{\mathcal{M}}_{bj}\mathsf{H}^{ij}_{(S)} + \widehat{\mathcal{M}}^{ij\mathbf{T}}_a\widehat{\mathcal{M}}_{bk}\mathsf{H}^k_{ij}\\
&\quad -\frac{1}{12\sqrt{2}}\epsilon_{ijklm}\widehat{\mathcal{M}}^{ij\mathbf{T}}_a\widehat{\mathcal{M}}^{rs}_b\mathsf{H}^{klm}_{rs}\\
&\quad -\sqrt{3}\left(\widehat{\mathcal{M}}^{\mathbf{T}}_a\widehat{\mathcal{M}}_{bm} + \frac{1}{24}\epsilon_{ijklm}\widehat{\mathcal{M}}^{ij\mathbf{T}}_a\widehat{\mathcal{M}}^{kl}_b\right)\mathsf{H}^m]\end{aligned} \tag{19}$$

where $f_{ab}^{(\overline{126})(+)} = \frac{1}{2}(f_{ab}^{(\overline{126})} + f_{ba}^{(\overline{126})})$.

5. Quartic Couplings of $SO(10)$

The technique discussed in Secs.3-4 can be used to compute the quartic couplings. For illustrative purposes we consider the simplest example with the superpotenial,

$$\mathsf{W} = \frac{1}{2}\Phi_U \mathbf{M}_{UU'}^{(1)} \Phi_{U'} + f_{ab}^{(1)} < \Psi_{(-)a}^* |B| \Psi_{(+)b} > l_U^{(1)} \Phi_U + .. \tag{20}$$

where the indices U, U' run over several Higgs representations of the same kind. $\mathbf{M}^{(1)}$ represents the mass matrices and $f^{(1)}$ are constants. We now eliminate Φ_U using the F-flatness condition: $\frac{\partial \mathsf{W}}{\partial \Phi_U} = 0$. This leads to[7]

$$\begin{aligned}
\mathsf{W}^{(16\times\overline{16})_1(16\times\overline{16})_1} &= 2\lambda_{ab,cd}^{(1)} < \widehat{\Psi}_{(-)a}^* |B| \widehat{\Psi}_{(+)b} >< \widehat{\Psi}_{(-)c}^* |B| \widehat{\Psi}_{(+)d} > \\
&= \frac{1}{2}\lambda_{ab,cd}^{(1)} [-\widehat{\mathcal{N}}_{aij}^{\mathbf{T}} \widehat{\mathcal{M}}_b^{ij} \widehat{\mathcal{N}}_{ckl}^{\mathbf{T}} \widehat{\mathcal{M}}_d^{kl} + 4\widehat{\mathcal{N}}_a^{i\mathbf{T}} \widehat{\mathcal{M}}_{bi} \widehat{\mathcal{N}}_{cjk}^{\mathbf{T}} \widehat{\mathcal{M}}_d^{jk} \\
&\quad -4\widehat{\mathcal{N}}_a^{i\mathbf{T}} \widehat{\mathcal{M}}_{bi} \widehat{\mathcal{N}}_c^{j\mathbf{T}} \widehat{\mathcal{M}}_{dj} + 4\widehat{\mathcal{N}}_a^{\mathbf{T}} \widehat{\mathcal{M}}_b \widehat{\mathcal{N}}_{cij}^{\mathbf{T}} \widehat{\mathcal{M}}_d^{ij} \\
&\quad -8\widehat{\mathcal{N}}_a^{\mathbf{T}} \widehat{\mathcal{M}}_b \widehat{\mathcal{N}}_c^{i\mathbf{T}} \widehat{\mathcal{M}}_{di} - 4\widehat{\mathcal{N}}_a^{\mathbf{T}} \widehat{\mathcal{M}}_b \widehat{\mathcal{N}}_c^{\mathbf{T}} \widehat{\mathcal{M}}_d]
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\lambda_{ab,cd}^{(1)} &= f_{ab}^{(1)} f_{cd}^{(1)} l_U^{(1)} \left[\widetilde{\mathbf{M}}^{(1)} \left\{\mathbf{M}^{(1)} \widetilde{\mathbf{M}}^{(1)} - \mathbf{1}\right\}\right]_{UU'} l_{U'}^{(1)} \\
\widetilde{\mathbf{M}}^{(\cdot)} &= \left[\mathbf{M}^{(\cdot)} + \left(\mathbf{M}^{(\cdot)}\right)^{\mathbf{T}}\right]^{-1}
\end{aligned} \tag{22}$$

Similarly a complete determination of $(16 \times \overline{16})_{45}(16 \times \overline{16})_{45}$ and of $(16 \times \overline{16})_{210}(16 \times \overline{16})_{210}$ can be given[7].

6. Conclusion

In this paper we have given a brief overview of the $SO(2N)$ (N odd) invariant couplings. In Sec.2 we gave a brief summary of some of the salient features of $SO(2N)$ algebra in terms of oscillator modes. We exhibited the form of $SO(2N)$ invariant cubic couplings and then specialized these results to the $SO(10)$ case. In Sec.3 we introduced a basis involving reducible $SU(N)$ tensors in terms of which $SO(2N)$ invariant couplings have a simple expansion. This result is codified in the so called Basic Theorem which is stated at the end of Sec.3. In Sec.4 we used the Basic Theorem to decompose $SO(10)$ invariant cubic couplings in terms of $SU(5) \times U(1)$ invariant forms. An application of the Basic Theorem was given through two illustrative examples involving the 16 plet spinor representations and the tensor representations, 45 and $\overline{126}$. The analysis presented here should be of interest to model builders using $SO(2N)$ (N odd) type gauge groups.

7. Acknowledgments

This paper is dedicated to Prof. Pran Nath on the occasion of his sixty fifth birthday and for proceedings of NathFest. This research was supported in part by NSF grant PHY-0139967.

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