TELEPORTATION OF CAVITY FIELD STATES VIA CAVITY QED

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October 31, 2004

Abstract

In this article we discuss two schemes of teleportation of cavity field states. In the first scheme we consider cavities prepared in a coherent state and in the second scheme we consider cavities prepared in a superposition of zero and one Fock states.

PACS: 03.65.Ud; 03.67.Mn; 32.80.-t; 42.50.-p
Keywords: teleportation; entanglement; non-locality; Bell states; cavity QED.

1 INTRODUCTION

Quantum information and quantum computation opened a completely new prospect in information processing and are important and active fields of research [1, 2, 3]. Teleportation, proposed by Bennett *et al* [4], has important applications in quantum information and quantum computation [1]. The two ingredients which are essential in teleportation are the superposition principle and entanglement and its consequences non-locality. In teleportation a party, Alice, wants to transfer the unknown quantum state of a given system which someone gives to her, to a system with another party, Bob, which in principle, is far apart from Alice. In order to do that, Alice and Bob share a Bell state [1, 3] (or EPR state [5]) in which half of the Bell pair is with Alice and the other half is with Bob and follow a given prescription communicating classically with each other. In the end of the process Bob gets a state identical to the state of the original state in which Alice´s system was prepared and the state of Alice´s system is destroyed since according to the no-cloning theorem [6, 1] it is not possible to clone an arbitrary quantum state.

There has been a lot of theoretical proposals of schemes of teleportation. In Ref. [7] it is proposed a teleportation scheme based on cavity QED for the teleportation of an atomic state where the cavities are prepared in a entangled state of zero and one Fock states. In Ref. [8] it is proposed a scheme to teleport an atomic state for atoms in a cascade configuration making use of cavities prepared in a coherent state. In Ref. [9] it is proposed a scheme to teleport an atomic state for atoms in a lambda configuration making use of cavities prepared in a coherent state. In Ref. [10] it is presented a scheme of teleportation where a superposition of zero and one Fock states is teleported via cavity QED using atoms in a lambda configuration. An interesting proposition of generating EPR states and realization of teleportation using a dispersive atom-field interaction is presented in [11]. Teleportation has already been realized experimentally. It has been demonstrated using optical systems [12] and NMR [13].

In this article we consider Rydberg atoms [14] interacting with a superconducting cavity [15, 16]. We study two schemes of performing teleportation. In the first scheme we consider cavities prepared in a coherent state and in the second scheme we consider cavities prepared in a superposition of a zero and a one Fock states.

2 SCHEME 1

Consider a three-level cascade atom Ak with $\mid e_k\rangle, \mid f_k\rangle$ and $\mid g_k\rangle$ being the upper, intermediate and lower atomic state respectively (see Fig. 1). We assume that the transition $\mid f_k\rangle \rightleftharpoons \mid e_k\rangle$ is far enough from resonance with the cavity central frequency such that only virtual transitions occur between these states (only these states interact with the cavity field). In addition we assume that the transition $\mid e_k\rangle \rightleftharpoons \mid g_k\rangle$ is highly detuned from the cavity frequency so that there will be no coupling with the cavity field. Here we are going to consider only the effect of levels $\mid f_k\rangle$ and $\mid g_k\rangle$. We do not consider level $\mid e_k\rangle$ since it will not play any role in our scheme. Therefore, we have effectively a two-level system involving states $\mid f_k\rangle$ and $|g_k\rangle$. Considering levels $\mid f_k\rangle$ and $\mid g_k\rangle$, we can write an effective time evolution operator

$$U_k(t) = e^{i\varphi a^\dagger a} \mid f_k\rangle\langle f_k \mid + |g_k\rangle\langle g_k \mid, \tag{2.1}$$

where the second term above was put by hand just in order to take into account the effect of level $\mid g_k\rangle$. In (2.1) a ($a^\dagger$) is the annihilation (creation) operator for the field in the cavity, $\varphi = g^2\tau/\ \Delta$, g is the coupling constant, $\Delta = \omega_e - \omega_f - \omega$ is the detuning where ω_e and ω_f are the frequencies of the upper and intermediate levels respectively and ω is the cavity field frequency and τ is the atom-field interaction time. Let us take $\varphi = \pi$. Now, let us assume that we let atom $A1$ to interact with cavity C prepared in a coherent state. Let us define

$$\mid \psi_x, \pm\rangle_{Ak} = \frac{1}{\sqrt{2}}(\mid f_k\rangle \pm \mid g_k\rangle), \tag{2.2}$$

and let us assume that atom $A1$ is prepared in a Ramsey cavity $R1$ in the state $\mid \psi_x, +\rangle_{A1}$. Now, we assume that we let atom $A1$ to interact with cavity $C1$ prepared in the coherent state $|-\alpha\rangle_1$. Then, taking into account (2.1), the system $A1 - C1$ evolves to

$$\mid \psi\rangle_{A1-C1} = \frac{1}{\sqrt{2}}(\mid f_1\rangle|\alpha\rangle_1 + \mid g_1\rangle|-\alpha\rangle_1). \tag{2.3}$$

If we define the even and odd coherent states

$$\begin{aligned} |+\rangle_{Ck} &= \frac{1}{\sqrt{N_k^+}}(|\alpha\rangle_k + |-\alpha\rangle_k), \\ |-\rangle_{Ck} &= \frac{1}{\sqrt{N_k^-}}(|\alpha\rangle_k - |-\alpha\rangle_k), \end{aligned} \tag{2.4}$$

with $N_k^{\pm} = 2\left(1 \pm e^{-2|\alpha|^2}\right) \cong 2$ and ${}_{Ck}\langle + \mid - \rangle_{Ck} = 0$ [17] we have

$$\mid \psi\rangle_{A1-C1} = \frac{1}{2}[|+\rangle_{C1}(\mid f_1\rangle + \mid g_1\rangle) + |-\rangle_{C1}(\mid f_1\rangle - \mid g_1\rangle)]. \tag{2.5}$$

Making use of (2.2) we can rewrite the above expression as

$$\mid \psi\rangle_{A1-C1} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|\psi_x, +\rangle_{A1} + |-\rangle_{C1}|\psi_x, -\rangle_{A1}). \tag{2.6}$$

Now we let atom $A1$ to fly through another cavity $C2$ prepared in the coherent state $|-\alpha\rangle_2$ and we have

$$\mid \psi\rangle_{A1-C1-C2} = \frac{1}{\sqrt{2}}[|+\rangle_{C1}(|+\rangle_{C2}|\psi_x, +\rangle_{A1} + |-\rangle_{C2}|\psi_x, -\rangle_{A1}) + |-\rangle_{C1}(|+\rangle_{C2}|\psi_x, -\rangle_{A1} + |-\rangle_{C2}|\psi_x, +\rangle_{A1})]. \tag{2.7}$$

If $A1$ enters a Ramsey cavity $R2$ where the atomic states are rotated according to

$$\begin{aligned} |\psi_x, +\rangle_{A1} &\longrightarrow \mid f_1\rangle \\ |\psi_x, -\rangle_{A1} &\longrightarrow \mid g_1\rangle \end{aligned}$$

and we detect $\mid f_1\rangle$, we get

$$\mid \Phi^+\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|+\rangle_{C2} + |-\rangle_{C1}|-\rangle_{C2}). \tag{2.8}$$

We can also prepare the states

$$\mid \Phi^-\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|+\rangle_{C2} - |-\rangle_{C1}|-\rangle_{C2}), \tag{2.9}$$

and

$$\mid \Psi^+\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|-\rangle_{C2} + |-\rangle_{C1}|+\rangle_{C2}), \tag{2.10}$$

and

$$\mid \Psi^-\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|-\rangle_{C2} - |-\rangle_{C1}|+\rangle_{C2}). \tag{2.11}$$

The states (2.8), (2.9), (2.10) and (2.11) are Bell states and form a Bell basis [1, 3].

Now let us assume that Alice keeps with her cavity $C2$ and Bob cavity $C1$. Then they separate and let us assume that, later on, Alice decides to teleport a state

$$\mid \psi\rangle_{C3} = \zeta|+\rangle_{C3} + \xi|-\rangle_{C3} \tag{2.12}$$

to Bob. Let us see how we can prepare a state like (2.12). Suppose we prepare cavity $C3$ initially in a coherent state $|-\alpha\rangle_3$. Then we prepare a two-level atom B, with $\mid f\rangle$ and $\mid g\rangle$ being the upper and lower state respectively, in a coherent superposition, sending B in the lower state $\mid g\rangle$ through a first Ramsey cavity $K1$ where the atomic states are rotated according to

$$K_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} c_g & c_f \\ -c_f & c_g \end{bmatrix}, \tag{2.13}$$

and we get

$$| \psi\rangle_B = c_f \mid f\rangle + c_g \mid g\rangle. \tag{2.14}$$

After that, B flies through cavity $C3$ and, taking into account the time evolution operator (2.1), after B pass through $C3$ the state of the system $B - C3$, for $\varphi = \pi$, is given by

$$| \psi\rangle_{B-C3} = c_f \mid f\rangle \mid \alpha\rangle_3 + c_g \mid g\rangle \mid -\alpha\rangle_{3.}$$

Then, we send B through a second Ramsey zone $K2$ where the atomic states are rotated according to

$$K_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -ie^{i\theta} \\ -ie^{-i\theta} & 1 \end{bmatrix}, \tag{2.15}$$

that is,

$$\begin{aligned} | \quad f\rangle &\rightarrow \frac{1}{\sqrt{2}}(| f\rangle - ie^{-i\theta} \mid g\rangle), \\ | \quad g\rangle &\rightarrow \frac{1}{\sqrt{2}}(-ie^{i\theta} \mid f\rangle + \mid g\rangle), \end{aligned} \tag{2.16}$$

and therefore, the state of the system $B - C3$ will be

$$\begin{aligned} |\psi\rangle_{B-C3} \quad = \quad & \frac{1}{\sqrt{2}}[(c_f - ie^{i\theta}c_g) \mid +\rangle_{C3} + (c_f + ie^{i\theta}c_g) \mid -\rangle_{C3}] \mid f\rangle \\ & + \frac{1}{\sqrt{2}}[(-ie^{-i\theta}c_f + c_g) \mid +\rangle_{C3}) + (-ie^{-i\theta}c_f - c_g) \mid -\rangle_{C3}] \mid g\rangle, \end{aligned}$$

Now, in order to obtain a state $|\psi\rangle_{C3}$ in cavity $C3$, we detect atom B in $\mid f\rangle$ or in $\mid g\rangle$. If we detect $\mid f\rangle$ we have $\zeta = (c_f - ie^{i\theta}c_g)/\sqrt{2}$ and $\xi = (c_f + ie^{i\theta}c_g)/\sqrt{2}$. If we detect $\mid g\rangle$ we have $\zeta = (-ie^{-i\theta}c_f + c_g)/\sqrt{2}$ and $\xi = (-ie^{-i\theta}c_f - c_g)/\sqrt{2}$.

Now let us see how Alice can teleprot the state (2.12) to Bob. First we write the state formed by the direct product of the Bell state and this unknown state $\mid \Phi^+\rangle_{C1-C2} \mid \psi\rangle_{C3}$, that is,

$$\begin{aligned} | \quad \psi\rangle_{C1-C2-C3} = \frac{1}{\sqrt{2}}[\zeta(| +\rangle_{C1} \mid +\rangle_{C2} \mid +\rangle_{C3} + \mid -\rangle_{C1} \mid -\rangle_{C2} \mid +\rangle_{C3}) + \\ \xi(\quad | \quad +\rangle_{C1} \mid +\rangle_{C2} \mid -\rangle_{C3} + \mid -\rangle_{C1} \mid -\rangle_{C2} \mid -\rangle_{C3})]. \end{aligned} \tag{2.17}$$

If Alice sends an atom $A2$ through $C2$ and $C3$ prepared initially in the state $|\psi_x, +\rangle_{A2}$ in a Ramsey cavity $R3$, we have

$$\begin{aligned} | \quad \psi\rangle_{C1-C2-C3-A2} = \frac{1}{\sqrt{2}}[\zeta(| +\rangle_{C1} \mid +\rangle_{C2} \mid +\rangle_{C3}|\psi_x, +\rangle_{A2} - \mid -\rangle_{C1} \mid -\rangle_{C2} \mid +\rangle_{C3}|\psi_x, -\rangle_{A2}) + \\ \xi(- \quad | \quad +\rangle_{C1} \mid +\rangle_{C2} \mid -\rangle_{C3}|\psi_x, -\rangle_{A2} + \mid -\rangle_{C1} \mid -\rangle_{C2} \mid -\rangle_{C3}|\psi_x, +\rangle_{A2})], \end{aligned} \tag{2.18}$$

which can be rewritten as

$$\begin{aligned} & | \quad \psi\rangle_{C1-C2-C3-A2} = \\ \frac{1}{\sqrt{2}}[\quad & | \quad \Phi^+\rangle_{C2-C3}(\zeta \mid +\rangle_{C1} + \xi \mid -\rangle_{C1})|\psi_x, +\rangle_{A2} + \\ & | \quad \Phi^-\rangle_{C2-C3}(\zeta \mid +\rangle_{C1} - \xi \mid -\rangle_{C1})|\psi_x, +\rangle_{A2} + \\ - \quad & | \quad \Psi^+\rangle_{C2-C3}(\zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1})|\psi_x, -\rangle_{A2} + \\ - \quad & | \quad \Psi^-\rangle_{C2-C3}(-\zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1})|\psi_x, -\rangle_{A2}]. \end{aligned} \tag{2.19}$$

Now Alice sends $A2$ through a Ramsey cavity $R4$ where the atomic states are rotated according to

$$\begin{aligned} |\psi_x, +\rangle_{A2} &\longrightarrow \mid f_2\rangle, \\ |\psi_x, -\rangle_{A2} &\longrightarrow \mid g_2\rangle, \end{aligned}$$

and if she detects $\mid f_2\rangle$ she gets

$$\mid \psi\rangle_{C1-C2-C3} = \frac{1}{\sqrt{2}}[\mid \Phi^+\rangle_{C2-C3}(\zeta \mid +\rangle_{C1} + \xi \mid -\rangle_{C1}) + \mid \Phi^-\rangle_{C2-C3}(\zeta \mid +\rangle_{C1} - \xi \mid -\rangle_{C1})], \quad (2.20)$$

and if she detects $\mid g_2\rangle$ she gets

$$\mid \psi\rangle_{C1-C2-C3} = \frac{1}{\sqrt{2}}[\mid \Psi^+\rangle_{C2-C3}(\zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1}) + \mid \Psi^-\rangle_{C2-C3}(-\zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1})]. \quad (2.21)$$

Notice that

$$\begin{aligned} \mid \Phi^+\rangle_{C2-C3} &= \frac{1}{\sqrt{2}}(|\alpha\rangle_2|\alpha\rangle_3 + |-\alpha\rangle_2|-\alpha\rangle_3), \\ \mid \Phi^-\rangle_{C2-C3} &= \frac{1}{\sqrt{2}}(|\alpha\rangle_2|-\alpha\rangle_3 + |-\alpha\rangle_2|\alpha\rangle_3), \\ \mid \Psi^+\rangle_{C2-C3} &= \frac{1}{\sqrt{2}}(|\alpha\rangle_2|\alpha\rangle_3 - |-\alpha\rangle_2|-\alpha\rangle_3), \\ \mid \Psi^-\rangle_{C2-C3} &= \frac{1}{\sqrt{2}}(|-\alpha\rangle_2|\alpha\rangle_3 - |\alpha\rangle_2|-\alpha\rangle_3). \end{aligned} \quad (2.22)$$

Now Alice injects $|\alpha\rangle_2$ or $|-\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ or $|-\alpha\rangle_3$ in $C3$. Then Alice sends a two-level atom $A3$ resonant with the cavity $C2$, with $|b_3\rangle$ and $|a_3\rangle$ being the lower and upper levels respectively, through $C2$ and a two-level atom $A4$ resonant with the cavity $C3$, with $|b_4\rangle$ and $|a_4\rangle$ being the lower and upper levels respectively, through $C3$. If Aj is sent in the lower state $|b_j\rangle$, under the Jaynes-Cummings dynamics [18, 19] we know that the state $|b_j\rangle|0\rangle_k$ ($j = 3, 4$ and $k = 2, 3$) does not evolve, however, the state $|b_j\rangle|\pm 2\alpha\rangle_k$ evolves to $|a_j\rangle|\chi_a^{\pm}\rangle_k + |b_j\rangle|\chi_b^{\pm}\rangle_k$, where $|\chi_b^{\pm}\rangle_k = \sum_n C_n^{\pm}\cos(gt\sqrt{n})|n\rangle_k$ and $|\chi_a^{\pm}\rangle_k = -i\sum_n C_{n+1}^{\pm}\sin(gt\sqrt{n+1})|n\rangle_k$ and $C_n^{\pm} = e^{-\frac{1}{2}|\pm 2\alpha_k|^2}(\pm 2\alpha_k)^n/\sqrt{n!}$. Therefore, the injection of $|\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ or the injection of $|-\alpha\rangle_2$ in $C2$ and $|-\alpha\rangle_3$ in $C3$ and the detection of $|a_3\rangle$ and $|a_4\rangle$ corresponds to the detection of $\mid \Phi^+\rangle_{C2-C3}$ or $\mid \Psi^+\rangle_{C2-C3}$. The injection of $|\alpha\rangle_2$ in $C2$ and $|-\alpha\rangle_3$ in $C3$ or the injection of $|-\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ and the detection of $|a_3\rangle$ and $|a_4\rangle$ corresponds to the detection of $\mid \Phi^-\rangle_{C2-C3}$ or $\mid \Psi^-\rangle_{C2-C3}$. Therefore, if Alice detects $\mid f_2\rangle$ and injects $|\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ or $|-\alpha\rangle_2$ in $C2$ and $|-\alpha\rangle_3$ in $C3$ and detects $|a_3\rangle$ and $|a_4\rangle$, Bob gets

$$\mid \psi\rangle_{C1} = \zeta \mid +\rangle_{C1} + \xi \mid -\rangle_{C1}. \quad (2.23)$$

If Alice detects $\mid f_2\rangle$ and injects $|\alpha\rangle_2$ in $C2$ and $|-\alpha\rangle_3$ in $C3$ or $|-\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ and detects $|a_3\rangle$ and $|a_4\rangle$, Bob gets

$$\mid \psi\rangle_{C1} = \zeta \mid +\rangle_{C1} - \xi \mid -\rangle_{C1}. \quad (2.24)$$

If Alice detects $\mid g_2\rangle$ and injects $|\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ or $|-\alpha\rangle_2$ in $C2$ and $|-\alpha\rangle_3$ in $C3$ and detects $|a_3\rangle$ and $|a_4\rangle$, Bob gets

$$\mid \psi\rangle_{C1} = \zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1}. \quad (2.25)$$

If Alice detects $| g_2\rangle$ and injects $|\alpha\rangle_2$ in $C3$ and $|-\alpha\rangle_3$ in $C3$ or $|-\alpha\rangle_2$ in $C2$ and $|\alpha\rangle_3$ in $C3$ and detects $|a_3\rangle$ and $|a_4\rangle$, Bob gets

$$| \psi\rangle_{C1} = -\zeta \mid -\rangle_{C1} + \xi \mid +\rangle_{C1}. \tag{2.26}$$

Notice that in the case of (2.23) Bob gets the right state and he has to do nothing else. In the case of (2.24) Bob can prepare an atom $A5$ in a Ramsey cavity $R5$ in the state

$$| \psi\rangle_{A5} = \frac{1}{\sqrt{2}}(| f_5\rangle + | g_5\rangle). \tag{2.27}$$

and send $A5$ through $C1$. After $A5$ fly through $C1$ Bob gets

$$| \psi\rangle_{C1-A5} = \frac{1}{\sqrt{2}}[\zeta(| f_5\rangle + | g_5\rangle) \mid +\rangle_{C1} - \xi(- | f_5\rangle + | g_5\rangle) \mid -\rangle_{C1}], \tag{2.28}$$

and if he detects $| f_5\rangle$ he gets the right state (2.23). In the case of (2.25) and (2.26) it is not possible to fix the states and Bob cannot do anything to get the correct teleported state. In Fig. 2 we present the setup of the above teleportation experiment.

3 SCHEME 2

We start assuming that we have a cavity Ck prepared in the state

$$|+\rangle_{Ck} = \frac{(|0\rangle_k + |1\rangle_k)}{\sqrt{2}}. \tag{3.29}$$

In order to prepare this state, we send a two-level atom $A0$, with $|f_0\rangle$ and $|e_0\rangle$ being the lower and upper level respectively, in the state

$$| \psi\rangle_{A0} = \frac{1}{\sqrt{2}}(i \mid e_0\rangle + | f_0\rangle), \tag{3.30}$$

through Ck, for $A0$ resonant with the cavity. If g is the coupling constant and τ the atom-field interaction time, under the Jaynes-Cummings dynamics, for $g\tau = \pi/2$, we know that the state $|f_0\rangle|0\rangle_k$ does not evolve, however, the state $|e_0\rangle|0\rangle_k$ evolves to $-i|f_0\rangle|1\rangle_k$. Then, for the cavity initially in the vacuum state $|0\rangle_k$, we have

$$\frac{(|f_0\rangle + i|e_0\rangle)}{\sqrt{2}}|0\rangle_k \longrightarrow |f_0\rangle\frac{(|0\rangle_k + |1\rangle_k)}{\sqrt{2}} = |f_0\rangle|+\rangle_{Ck} \tag{3.31}$$

If we start with

$$| \psi\rangle_{A0} = \frac{1}{\sqrt{2}}(-i \mid e_0\rangle + | f_0\rangle), \tag{3.32}$$

we get

$$|-\rangle_{Ck} = \frac{(|0\rangle_k - |1\rangle_k)}{\sqrt{2}}. \tag{3.33}$$

Now let us assume that cavities $C1$ and $C2$ are prepared in the state (3.29). Consider an atom $A1$ prepared in the state $| \psi_x, +\rangle_{A1}$ (see (2.2)) in a Ramsey cavity $R1$. Taking into account (2.1), after atom $A1$ has passed through the cavities, we get

$$| \psi\rangle_{A1-C1-C2} = \frac{1}{\sqrt{2}}(|-\rangle_{C1}|-\rangle_{C2} \mid f_1\rangle + |+\rangle_{C1}|+\rangle_{C2} \mid g_1\rangle), \tag{3.34}$$

Now, if atom $A1$ enters a second Ramsey cavity $R2$ where the atomic states are rotated according to

$$
\begin{aligned}
\mid \ f_1\rangle &\rightarrow \frac{1}{\sqrt{2}}(\mid f_1\rangle + \mid g_1\rangle), \\
\mid \ g_1\rangle &\rightarrow \frac{1}{\sqrt{2}}(- \mid f_1\rangle + \mid g_1\rangle),
\end{aligned}
\tag{3.35}
$$

after we detect $\mid g_1\rangle$ we have

$$
\mid \Phi^+\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|+\rangle_{C2} + |-\rangle_{C1}|-\rangle_{C2}), \tag{3.36}
$$

It is also easy to prepare

$$
\mid \Phi^-\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|+\rangle_{C2} - |-\rangle_{C1}|-\rangle_{C2}), \tag{3.37}
$$

and

$$
\mid \Psi^+\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|-\rangle_{C2} + |-\rangle_{C1}|+\rangle_{C2}), \tag{3.38}
$$

and

$$
\mid \Psi^-\rangle_{C1-C2} = \frac{1}{\sqrt{2}}(|+\rangle_{C1}|-\rangle_{C2} - |-\rangle_{C1}|+\rangle_{C2}). \tag{3.39}
$$

The states (3.36), (3.37), (3.38) and (3.39) are Bell states and form a Bell basis [1, 3].

Now let us assume that Alice keeps with her cavity $C2$ and Bob cavity $C1$. Then they separate and let us assume that, later on, Alice decides to teleport a state

$$
\mid \psi\rangle_{C3} = \zeta|+\rangle_{C3} + \xi|-\rangle_{C3} \tag{3.40}
$$

to Bob. Now, let us see how we can prepare the state (3.40). First we send a two-level atom B, with $|f\rangle$ and $|e\rangle$ being the lower and upper level respectively, through a Ramsey cavity $K1$ in the lower state $|f\rangle$ where the atomic states are rotated according to

$$
K_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} c_f & c_e \\ -c_e & c_f \end{bmatrix}, \tag{3.41}
$$

and we get

$$
\mid \psi\rangle_B = c_e \mid e\rangle + c_f \mid f\rangle. \tag{3.42}
$$

Next we send B through $C3$ prepared in the vacuum state $|0\rangle_3$. If g is the coupling constant and τ the atom-field interaction time, under the Jaynes-Cummings dynamics, for $g\tau = \pi/2$, we know that the state $|f\rangle|0\rangle_3$ does not evolve, however, the state $|e\rangle|0\rangle_3$ evolves to $-i|f\rangle|1\rangle_3$. Then we have

$$
\mid \psi\rangle_{B-C3} = (c_f|0\rangle_3 - ic_e|1\rangle_3) \mid f\rangle. \tag{3.43}
$$

Therefore, making use of (3.29) and (3.33) the above state can be written as (3.40) with $\zeta = (c_f - ic_e)/\sqrt{2}$ and $\xi = (c_f + ic_e)/\sqrt{2}$.

Now let us see how Alice can teleprot the state (3.40) to Bob. First we write the state formed by the direct product of the Bell state and this unknown state $\mid \Phi^+\rangle_{C1-C2} \mid \psi\rangle_{C3}$, that is,

$$
\begin{aligned}
\mid \ \psi\rangle_{C1-C2-C3} = \frac{1}{\sqrt{2}}[\zeta(\mid +\rangle_{C1} \mid +\rangle_{C2} \mid +\rangle_{C3} + \mid -\rangle_{C1} \mid -\rangle_{C2} \mid +\rangle_{C3}) + \\
\xi(\ \mid \ +\rangle_{C1} \mid +\rangle_{C2} \mid -\rangle_{C3} + \mid -\rangle_{C1} \mid -\rangle_{C2} \mid -\rangle_{C3})].
\end{aligned}
\tag{3.44}
$$

Now Alice prepares an atom $A2$ in the state $|\psi_x,+\rangle_{A2}$ in a Ramsey cavity $R3$ and send it through cavities $C2$ and $C3$ and we have

$$\begin{aligned}
&|\psi\rangle_{C1-C2-C3-A2} = \\
\frac{1}{\sqrt{2}}\{\zeta[&|+\rangle_{C1}(|-\rangle_{C2}|-\rangle_{C3}|f_2\rangle+|+\rangle_{C2}|+\rangle_{C3}|g_2\rangle)+ \\
&|-\rangle_{C1}(|+\rangle_{C2}|-\rangle_{C3}|f_2\rangle+|-\rangle_{C2}|+\rangle_{C3}|g_2\rangle)]+ \qquad (3.45)\\
\xi[&|+\rangle_{C1}(|-\rangle_{C2}|+\rangle_{C3}|f_2\rangle+|+\rangle_{C2}|-\rangle_{C3}|g_2\rangle)+ \\
&|-\rangle_{C1}(|+\rangle_{C2}|+\rangle_{C3}|f_2\rangle+|-\rangle_{C2}|-\rangle_{C3}|g_2\rangle)]\} \qquad (3.46)
\end{aligned}$$

Then Alice sends $A2$ through a Ramsey cavity $R4$ where the atomic states are rotated according to

$$\begin{aligned}
|f_2\rangle &\rightarrow \frac{1}{\sqrt{2}}(|f_2\rangle+|g_2\rangle), \\
|g_2\rangle &\rightarrow \frac{1}{\sqrt{2}}(-|f_2\rangle+|g_2\rangle), \qquad (3.47)
\end{aligned}$$

and we have

$$\begin{aligned}
&|\psi\rangle_{C1-C2-C3-A2} = \frac{1}{2}\{\zeta[|+\rangle_{C1}(|\Phi^+\rangle_{C2-C3}|g_2\rangle-|\Phi^-\rangle_{C2-C3}|f_2\rangle)+ \\
&|-\rangle_{C1}(|\Psi^+\rangle_{C2-C3}|g_2\rangle+|\Psi^-\rangle_{C2-C3}|f_2\rangle)]+ \\
\xi[&|+\rangle_{C1}(|\Psi^+\rangle_{C2-C3}|g_2\rangle-|\Psi^-\rangle_{C2-C3}|f_2\rangle)+ \\
&|-\rangle_{C1}(|\Phi^+\rangle_{C2-C3}|g_2\rangle+|\Phi^-\rangle_{C2-C3}|f_2\rangle)]\}. \qquad (3.48)
\end{aligned}$$

If Alice detects $|g_2\rangle$ she gets

$$\begin{aligned}
&|\psi\rangle_{C1-C2-C3-A2} = \frac{1}{2}[\zeta(|+\rangle_{C1}|\Phi^+\rangle_{C2-C3}+|-\rangle_{C1}|\Psi^+\rangle_{C2-C3}) \\
+\xi(&|+\rangle_{C1}|\Psi^+\rangle_{C2-C3}+|-\rangle_{C1}|\Phi^+\rangle_{C2-C3})], \qquad (3.49)
\end{aligned}$$

and if she detects $|f_2\rangle$ she gets

$$\begin{aligned}
&|\psi\rangle_{C1-C2-C3-A2} = \frac{1}{2}[\zeta(-|+\rangle_{C1}|\Phi^-\rangle_{C2-C3}+|-\rangle_{C1}|\Psi^-\rangle_{C2-C3})+ \\
\xi(-&|+\rangle_{C1}|\Psi^-\rangle_{C2-C3}+|-\rangle_{C1}|\Phi^-\rangle_{C2-C3})]. \qquad (3.50)
\end{aligned}$$

Notice that

$$|\Phi^+\rangle_{C2-C3} = \frac{1}{\sqrt{2}}(|0\rangle_{C2}|0\rangle_{C3}+|1\rangle_{C2}|1\rangle_{C3}), \qquad (3.51)$$

$$|\Phi^-\rangle_{C2-C3} = \frac{1}{\sqrt{2}}(|1\rangle_{C2}|0\rangle_{C3}+|0\rangle_{C2}|1\rangle_{C3}), \qquad (3.52)$$

$$|\Psi^+\rangle_{C2-C3} = \frac{1}{\sqrt{2}}(|0\rangle_{C2}|0\rangle_{C3}-|1\rangle_{C2}|1\rangle_{C3}), \qquad (3.53)$$

$$|\Psi^-\rangle_{C2-C3} = \frac{1}{\sqrt{2}}(|1\rangle_{C2}|0\rangle_{C3}-|0\rangle_{C2}|1\rangle_{C3}). \qquad (3.54)$$

Now Alice sends a two-level atom $A3$ through $C2$ and a two-level atom $A4$ through $C3$, both resonant with the respective cavity. Let $|f_3\rangle$ and $|e_3\rangle$ be the lower and upper level of $A3$ respectively and

$|f_4\rangle$ and $|e_4\rangle$ be the lower and upper level of $A4$ respectively. If g is the coupling constant and τ the atom-field interaction time, under the Jaynes-Cummings dynamics, for $g\tau = \pi/2$, we have

$$|f_3\rangle|f_4\rangle \quad | \quad \Phi^+\rangle_{C2-C3} \longrightarrow \frac{1}{\sqrt{2}}(|f_3\rangle|f_4\rangle - |e_3\rangle|e_4\rangle)|0\rangle_{C2}|0\rangle_{C3} = |\Phi^-\rangle_{A3-A4}|0\rangle_{C2}|0\rangle_{C3}, \tag{3.55}$$

$$|f_3\rangle|f_4\rangle \quad | \quad \Phi^-\rangle_{C2-C3} \longrightarrow -\frac{i}{\sqrt{2}}(|f_3\rangle|e_4\rangle + |e_3\rangle|f_4\rangle)|0\rangle_{C2}|0\rangle_{C3} = |\Psi^+\rangle_{A3-A4}|0\rangle_{C2}|0\rangle_{C3}, \tag{3.56}$$

$$|f_3\rangle|f_4\rangle \quad | \quad \Psi^+\rangle_{C2-C3} \longrightarrow \frac{1}{\sqrt{2}}(|f_3\rangle|f_4\rangle + |e_3\rangle|e_4\rangle)|0\rangle_{C2}|0\rangle_{C3} = |\Phi^+\rangle_{A3-A4}|0\rangle_{C2}|0\rangle_{C3}, \tag{3.57}$$

$$|f_3\rangle|f_4\rangle \quad | \quad \Psi^-\rangle_{C2-C3} \longrightarrow \frac{i}{\sqrt{2}}(|f_3\rangle|e_4\rangle - |e_3\rangle|f_4\rangle)|0\rangle_{C2}|0\rangle_{C3} = |\Psi^-\rangle_{A3-A4}|0\rangle_{C2}|0\rangle_{C3}. \tag{3.58}$$

Then, in the case that Alice had detected $| \, g_2\rangle$, we have

$$\begin{aligned} | \quad \psi\rangle_{C1-C2-C3-A2} &= \frac{1}{2}[\zeta(| +\rangle_{C1}|\Phi^-\rangle_{A3-A4} + | -\rangle_{C1}|\Phi^+\rangle_{A3-A4}) \\ +\xi(\quad | \quad +\rangle_{C1}|\Phi^+\rangle_{A3-A4} &+ | -\rangle_{C1}|\Phi^-\rangle_{A3-A4})], \end{aligned} \tag{3.59}$$

and if she had detected $| \, f_2\rangle$, we have

$$\begin{aligned} | \quad \psi\rangle_{C1-C2-C3-A2} &= \frac{1}{2}[\zeta(- | +\rangle_{C1}|\Psi^+\rangle_{A3-A4} + | -\rangle_{C1}|\Psi^-\rangle_{A3-A4}) + \\ \xi(- \quad | \quad +\rangle_{C1}|\Psi^-\rangle_{A3-A4} &+ | -\rangle_{C1}|\Psi^+\rangle_{A3-A4})]. \end{aligned} \tag{3.60}$$

Now we define

$$\Sigma_x = \sigma_x^3\sigma_x^4, \tag{3.61}$$

where

$$\sigma_x^k =| \, f_k\rangle\langle e_k \, | + | \, e_k\rangle\langle f_k \, |, \tag{3.62}$$

and we have

$$\begin{aligned} \Sigma_x \quad | \quad \Psi^\pm\rangle_{A3-A4} &= \pm \, | \, \Psi^\pm\rangle_{A3-A4}, \\ \Sigma_x \quad | \quad \Phi^\pm\rangle_{A3-A4} &= \pm \, | \, \Phi^\pm\rangle_{A3-A4}. \end{aligned} \tag{3.63}$$

Therefore, we can distinguish between $(| \, \Psi^+\rangle_{A3-A4}, | \, \Phi^+\rangle_{A3-A4})$ and $(| \, \Psi^-\rangle_{A3-A4}, | \, \Phi^-\rangle_{A3-A4})$ performing measurements of $\Sigma_x = \sigma_x^3\sigma_x^4$. In order to do so, we proceed as follows. We make use of

$$K_k = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \tag{3.64}$$

or

$$K_k = \frac{1}{\sqrt{2}}(| \, f_k\rangle\langle f_k \, | - | \, f_k\rangle\langle e_k \, | + | \, e_k\rangle\langle f_k \, | + | \, e_k\rangle\langle e_k \, |), \tag{3.65}$$

to gradually unravel the Bell states. The eigenvectors of the operators σ_x^k are

$$|\psi_x, \pm\rangle_{Ak} = \frac{1}{\sqrt{2}}(| \, f_k\rangle \pm | \, e_k\rangle), \tag{3.66}$$

and we can rewrite the Bell states as

$$\begin{aligned} | \quad \Phi^\pm\rangle_{A3-A4} &= \frac{1}{2}[|\psi_x, +\rangle_{A3}(| \, f_4\rangle \pm | \, e_4\rangle) + |\psi_x, -\rangle_{A3}(| \, f_4\rangle \mp | \, e_4\rangle)], \\ | \quad \Psi^\pm\rangle_{A3-A4} &= \frac{1}{2}[|\psi_x, +\rangle_{A3}(| \, e_4\rangle \pm | \, f_4\rangle) + |\psi_x, -\rangle_{A3}(| \, e_4\rangle \mp | \, f_4\rangle)]. \end{aligned} \tag{3.67}$$

Let us take for instance (3.57),

$$| \Phi^+\rangle_{A3-A4} = \frac{1}{\sqrt{2}}(| f_3\rangle \ | f_4\rangle + | e_3\rangle \ | e_4\rangle). \quad (3.68)$$

Applying K_3 to this state we have

$$K_3 \ | \Phi^+\rangle_{A3-A4} = \frac{1}{2}[|f_3\rangle(| f_4\rangle - | e_4\rangle) + |e_3\rangle(| f_4\rangle + | e_4\rangle)]. \quad (3.69)$$

Now, we compare (3.69) and (3.67). We see that the rotation by K_3 followed by the detection of $|e_3\rangle$ corresponds to the detection of the the state $|\psi_x, +\rangle_{A3}$ whose eigenvalue of σ_x^3 is +1. After we detect $|e_3\rangle$, we get

$$| \psi\rangle_{A4} = \frac{1}{\sqrt{2}}(| f_4\rangle + | e_4\rangle), \quad (3.70)$$

that is, we have got

$$| \psi\rangle_{A4} = |\psi_x, +\rangle_{A4}. \quad (3.71)$$

If we apply (3.65) for $k = 4$ to the state (3.71) we get

$$K_4 \ | \psi\rangle_{A4} = |e_4\rangle. \quad (3.72)$$

We see that the rotation by K_4 followed by the detection of $|e_4\rangle$ corresponds to the detection of the the state $|\psi_x^4, +\rangle$ whose eigenvalue of σ_x^4 is +1. The same applies to (3.56).

Summarizing, we have two possible sequences of atomic state rotations through K_k and detections of $| f_k\rangle$ or $| e_k\rangle$ and the corresponding states $|\psi_x^k, \pm\rangle$ where $k = 3$ and 4 which corresponds to the measurement of the eigenvalue +1 of the operator Σ_x given by (3.63) and the detection of (3.57) or (3.56) corresponds to

$$\begin{aligned} (K_3, \ \ | \ \ e_3\rangle)(K_4, | e_4\rangle) &\longleftrightarrow |\psi_x, +\rangle_{A3}|\psi_x, +\rangle_{A4}, \\ (K_3, \ \ | \ \ f_3\rangle)(K_4, | f_4\rangle) &\longleftrightarrow |\psi_x, -\rangle_{A3}|\psi_x, -\rangle_{A4}. \end{aligned} \quad (3.73)$$

Considering (3.55) and (3.58) we have

$$\begin{aligned} (K_3, \ \ | \ \ e_3\rangle)(K_4, | f_4\rangle) &\longleftrightarrow |\psi_x, +\rangle_{A3}|\psi_x, -\rangle_{A4}, \\ (K_1, \ \ | \ \ f_3\rangle)(K_4, | e_4\rangle) &\longleftrightarrow |\psi_x, -\rangle_{A3}|\psi_x, +\rangle_{A4}, \end{aligned} \quad (3.74)$$

which corresponds to the measurement of the eigenvalue −1 of the operator Σ_x given by (3.63).

Therefore, after the sequence $| g_2\rangle(K_3, | e_3\rangle)(K_4, | e_4\rangle)$ or $| g_2\rangle(K_3, | f_3\rangle)(K_4, | f_4\rangle)$ Bob gets,

$$| \psi\rangle_{C1} = \zeta \ | -\rangle_{C1} + \xi \ | +\rangle_{C1}. \quad (3.75)$$

After the sequence $| g_2\rangle(K_3, | e_3\rangle)(K_4, | f_4\rangle)$ or $| g_2\rangle(K_3, | f_3\rangle)(K_4, | e_4\rangle)$ Bob gets,

$$| \psi\rangle_{C1} = \zeta \ | +\rangle_{C1} + \xi \ | -\rangle_{C1}. \quad (3.76)$$

After the sequence $| f_2\rangle(K_3, | e_3\rangle)(K_4, | e_4\rangle)$ or $| f_2\rangle(K_3, | f_3\rangle)(K_4, | f_4\rangle)$ Bob gets,

$$| \psi\rangle_{C1} = -\zeta \ | +\rangle_{C1} + \xi \ | -\rangle_{C1}. \quad (3.77)$$

After the sequence $| f_2\rangle(K_3, | e_3\rangle)(K_4, | f_4\rangle)$ or $| f_2\rangle(K_3, | f_3\rangle)(K_4, | e_4\rangle)$ Bob gets,

$$| \psi\rangle_{C1} = \zeta \ | -\rangle_{C1} - \xi \ | +\rangle_{C1}. \quad (3.78)$$

In the case of (3.76) Bob gets the right state and he has to do nothing else. In the case (3.75), consider a two-level atom $A5$ with $| e_5\rangle$ and $| f_5\rangle$ being the upper and lower atomic state respectively such that the transition $| f_5\rangle \rightleftharpoons | e_5\rangle$ is far enough from resonance with the cavity central frequency so that we have a dispersive atom-field interaction. Then the time evolution operator is given by

$$U(t) = e^{-i\varphi(a^\dagger a+1)} | e\rangle\langle e | + e^{i\varphi a^\dagger a} | f\rangle\langle f |, \tag{3.79}$$

where $\varphi = g^2\tau/\Delta$. Then, for

$$| \psi\rangle_{A5} = \frac{1}{\sqrt{2}}(| f_5\rangle + | e_5\rangle),$$

and for $\varphi = \pi$, we have

$$\begin{aligned} | \psi\rangle_{A5} | +\rangle_{C1} &\longrightarrow \frac{1}{\sqrt{2}}(- | f_5\rangle + | e_5\rangle) | -\rangle_{C1}, \\ | \psi\rangle_{A5} | -\rangle_{C1} &\longrightarrow \frac{1}{\sqrt{2}}(- | f_5\rangle + | e_5\rangle) | +\rangle_{C1}, \end{aligned}$$

and after sending atom $A5$ through $C1$ Bob gets the right state. Notice finally that it is not possible to fix the states (3.77) and (3.78). In Fig. 3 we present the setup of the above teleportation experiment.

4 CONCLUSION

In this article we have studied two schemes of teleportation of cavity field states by the interaction of Rydberg atoms with superconducting cavities. In the first scheme we show how to teleport a state which is a superposition of an even and an odd coherent state. In the second scheme the state to be teleported is a state constructed with zero and one Fock states. In both schemes it is possible to achieve teleportation only with 50% of success since it is not possible to handle cavity field states and to fix all the wrong teleported states as in the case of atomic states which can be rotated easily.

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